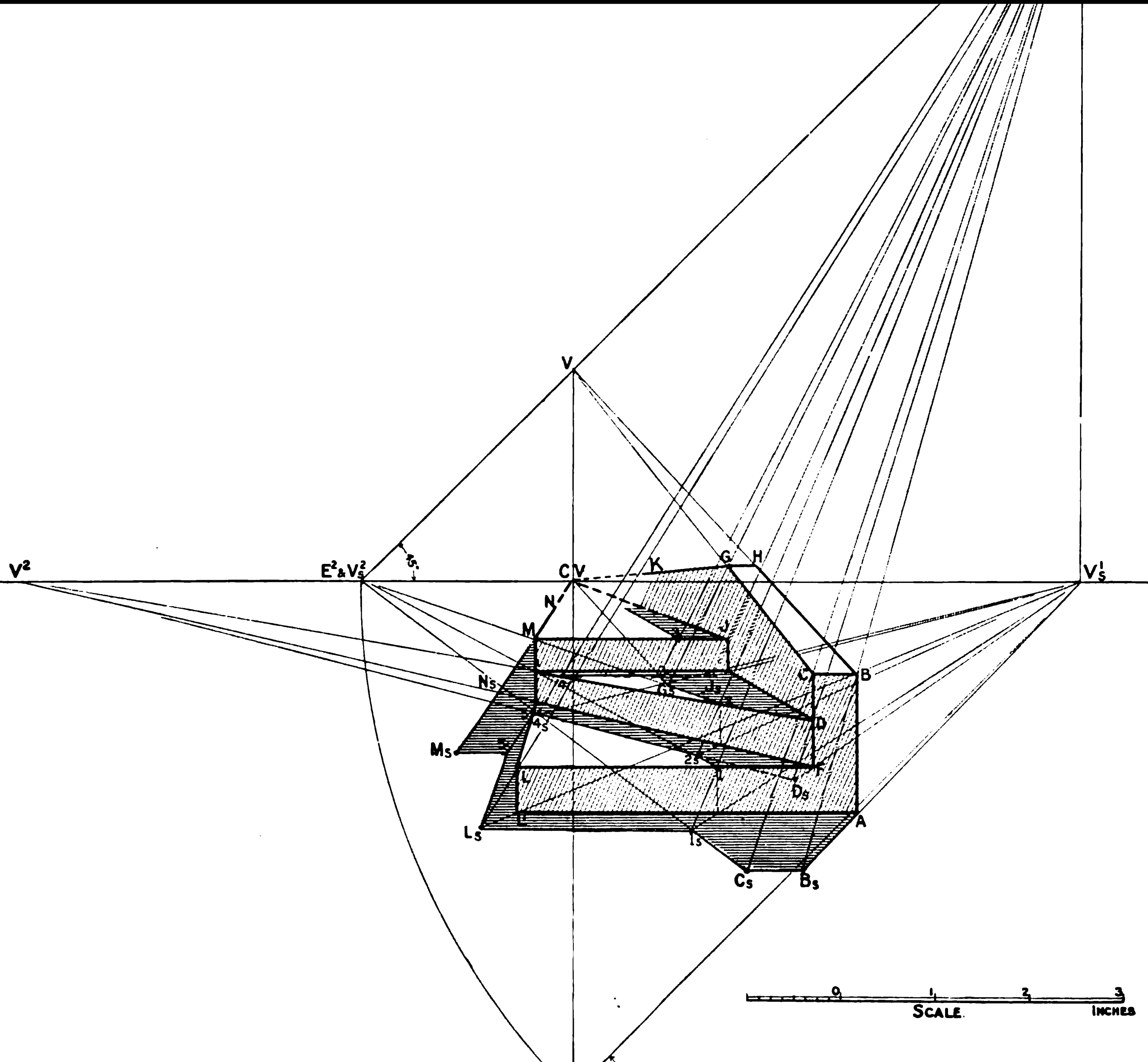

Advanced perspective

Lewes R. Crosskey, James Thaw

ADVANCED PERSPECTIVE

INVOLVING THE DRAWING OF
OBJECTS WHEN PLACED IN OBLIQUE POSITIONS, SHADOWS, AND REFLECTIONS

ARRANGED TO MEET THE REQUIREMENTS
OF ARCHITECTS, DRAUGHTSMEN, AND STUDENTS PREPARING FOR THE
PERSPECTIVE EXAMINATION OF THE EDUCATION DEPARTMENT

BY

LEWES R. CROSSKEY

ART MASTER, ALLAN GLEN'S SCHOOL; DIRECTOR OF THE DEPARTMENT OF
INDUSTRIAL ARTS, THE GLASGOW AND WEST OF SCOTLAND TECHNICAL COLLEGE

AND

JAMES THAW

ART MASTER AND MANUAL INSTRUCTOR, FALKIRK HIGH SCHOOL

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PREFACE.

This book is a continuation of my *Elementary Perspective,* and practically exhausts the subject. Students who have mastered these two books will find little difficulty in meeting any question set in Perspective. As the examinations on Perspective now require a very thorough knowledge of the general principles involved in the working out of a perspective drawing, great care has been taken in the present work to give full and ample statements regarding the principles underlying the various methods that are employed. It is not advisable for students to study this book without first mastering *Elementary Perspective,* as many explanations made in the first book are not repeated in the second.

LEWES R. CROSSKEY.

June, 1901.

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NOTATION.

P.P. The Picture Plane, *i.e.* an imaginary plane (generally vertical and transparent) situated between the spectator's eye and object, and upon which the drawing is supposed to be executed.

E. The Eye, *i.e.* the point from which the spectator views the object. When the eye is rotated into the **P.P.** about any line its position is then indicated as E^1, E^2, E^3, &c.

V.L. The Vanishing Line of a plane, *i.e.* the perspective representation of the line in which a plane would seem to disappear on being produced infinitely.

P.L. The Picture Line of a plane, *i.e.* the line in which a plane cuts the **P.P.**

H.L. The Horizontal Line, *i.e.* the **V.L.** of the ground and all horizontal planes.

G.L. Ground Line, *i.e.* the intersection of the ground with the **P.P.**

C.V. The Centre of Vision, *i.e.* the point on the **P.P.** opposite the spectator.

C.V.L. The Centre of the Vanishing Line, *i.e.* the point where a perpendicular from the eye to a **V.L.** cuts that **V.L.**

V. or **V.P.** Vanishing Point, *i.e.* the perspective representation of the point in which a line seems to disappear, on being produced infinitely, indicated thus: V^1, V^2, V^3, &c.

M. or **M.P.** Measuring Point, *i.e.* a **V.P.** of a line by means of which lengths can be measured off on other lines which are drawn in perspective. M^1 stands for the **M.P.** of V^1, M^2 corresponding to V^2, &c.

V^P and M^P. The **V.P.** and **M.P.** respectively of the perpendiculars to a plane.

D.P. Distance Point, *i.e.* the **M.P.** of lines vanishing at the **C.V.**

V_S. The **V.P.** of a line's shadow. When more than one are used they are indicated thus: V^1_S, V^2_S, &c.

A_S. The shadow of **A** on any plane. Similarly B_S, C_S, &c., are the shadows of **B**, **C**, &c.

V_R. The **V.P.** of a line's reflection. The reflection of V^1, V^2, V^3, &c., are denoted by V^1_R, V^2_R, V^3_R, &c.

S. The **V.P.** of the sun's rays.

H.T. The Horizontal Trace of a plane, *i.e.* the line in which a plane cuts the ground; when more than one is marked on a drawing they are denoted thus **H.T.**, **H.T.**^{2}, **H.T.**^{3}, &c. (**H.T.**s are generally shown by chain lines.)

ADVANCED PERSPECTIVE.

PLANES.

In obtaining the perspective representation of an object, the points and lines composing it are considered to lie in planes.

A plane has unlimited length and breadth, but no thickness.

A flat plane is one such that if any two points be taken on the plane the straight line joining them lies wholly on the plane. The word plane is used to denote a flat plane unless stated to the contrary.

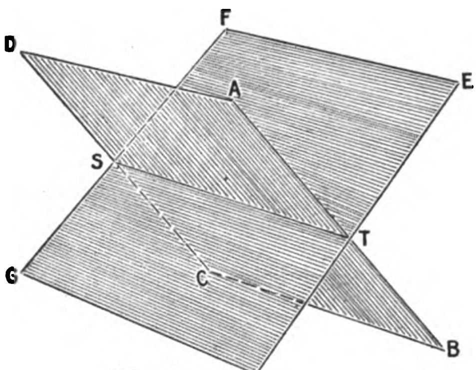


Fig. 1.

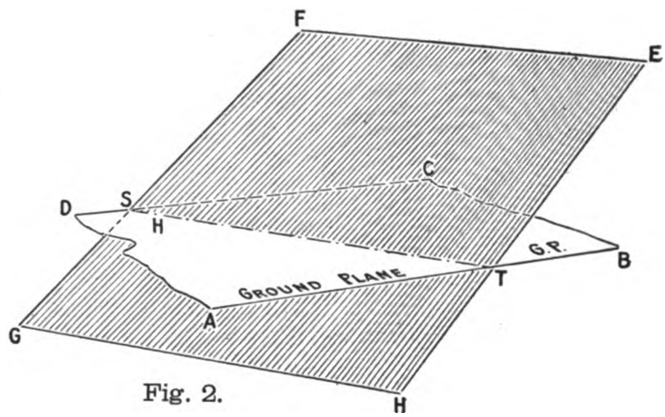


Fig. 2.

The intersection of any two planes is a straight line. In fig. 1 **ABCD** and **EFGH** are any two planes; their intersection is the *straight* line **ST**.

The line of intersection of a plane with the ground is called the *horizontal trace* (**H.T.**) of that plane. (In the present work horizontal traces are denoted by chain lines thus, ·—·—·—·.) In fig. 2 **ABCD** represents a portion of the ground plane. **ST** is the horizontal trace (**H.T.**) of the plane **EFGH**.

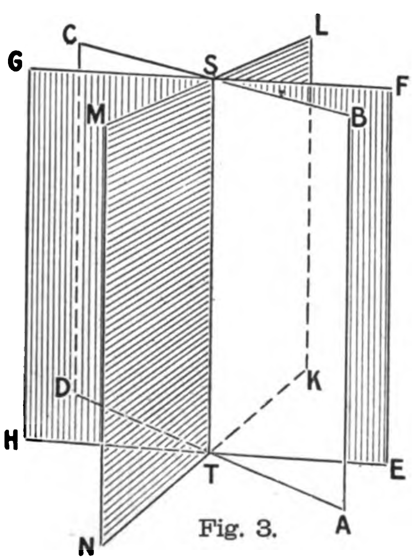


Fig. 3.

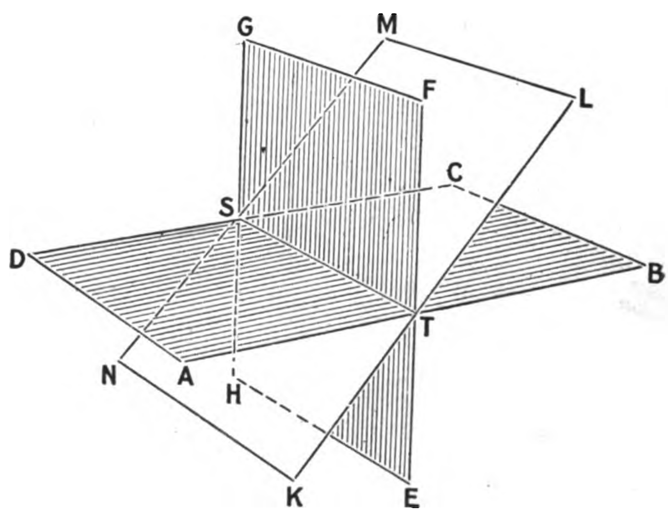


Fig. 4.

A vertical line can only be contained by a vertical plane. In fig. 3 **ABCD**, **EFGH** and **KLMN** are vertical planes containing **ST**.

An infinite number of *variously inclined* planes can contain a line (unless the line is vertical). In fig. 4 **ABCD**, **EFGH** and **KLMN** are variously inclined planes containing **ST**.

A vertical plane can contain any straight line. The plan of the line is on the **H.T.** of the plane. In fig. 5 **K L M N** is a vertical plane containing **A B**. **a b** on **K N** is the plan of **A B**.

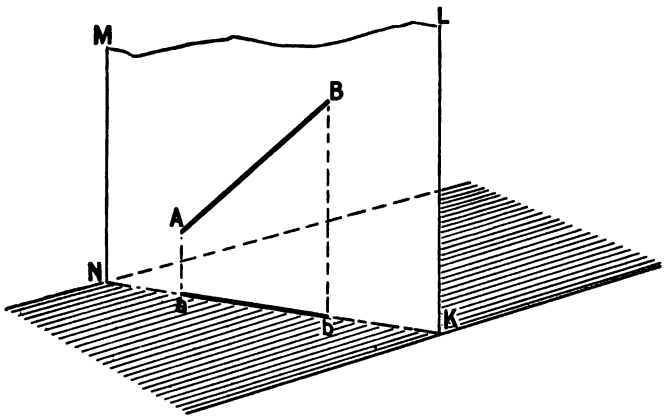


Fig. 5.

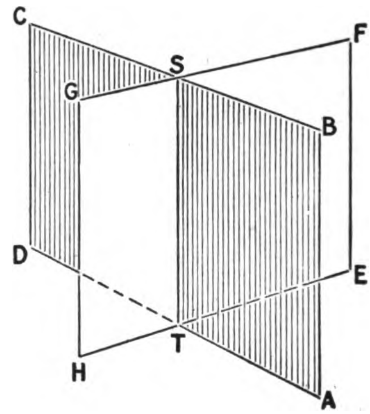


Fig. 6.

The intersection of two vertical planes is a vertical line. In fig. 6 **A B C D** and **E F G H** are vertical planes intersecting in the vertical line **S T**.

One plane can be found that will contain any three points not in one straight line, or a plane can contain any two intersecting lines. Referring to fig. 7 an indefinite number of planes, of which **E F G H** and **K L M N** are examples, can contain two points as **A** and **B**, but **E F G H** is the only plane that will contain the points **A B** and **C**.

If two planes intersect and any line be drawn on one of them, the point in which this line cuts the intersection of the planes is the intersection of the line with the other plane. In fig. 8 **A B C D** is the ground plane; **E F G H** and **K L M N** are other two planes, **E F** and **K N** being their **H.T.s.** **S T** is the intersection of **E F G H** and **K L M N**. **P Q** lies on **K L M N**; it intersects the ground on the **H.T.** of the plane at **R**. It intersects **E F G H** on **S T** at **O**.

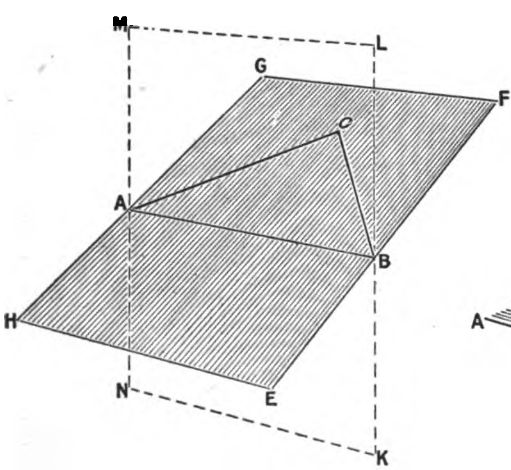


Fig. 7.

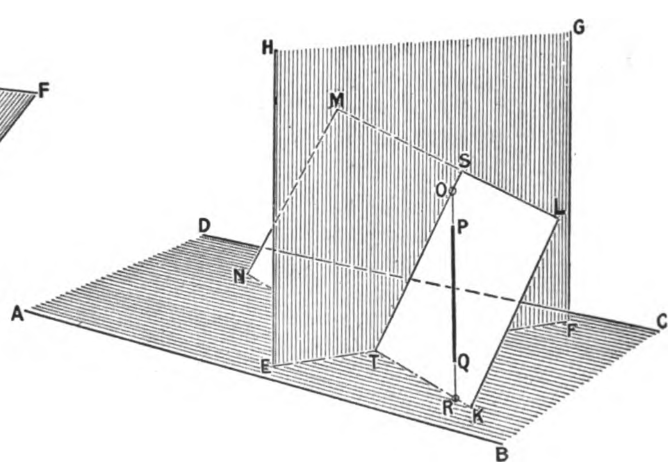


Fig. 8.

If a vertical plane intersects any other plane so that their **H.T.**s are at right angles, the intersection of the two planes is perpendicular to the **H.T.** of the second plane. (N.B. This line lies in both planes.) In fig. 9 **A B C D** represents the ground. **K L M N** is any other plane. **E F G H** is a vertical plane having its **H.T. E F** at right angles to **M N**, the **H.T.** of **K L M N**. **N K**, the intersection of **E F G H** and **K L M N**, is perpendicular to **M N**. Observe that the ∠ **K N F** determines the inclination of **K L M N** to the grounds.

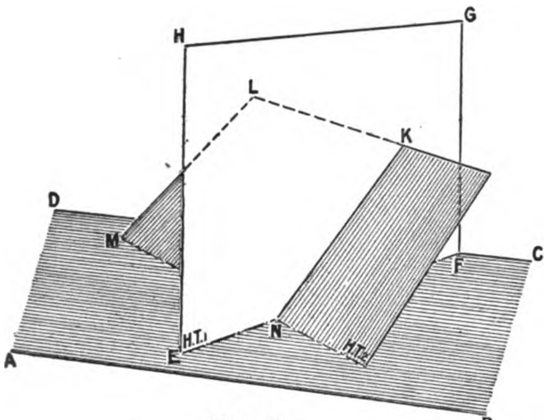


Fig. 9.

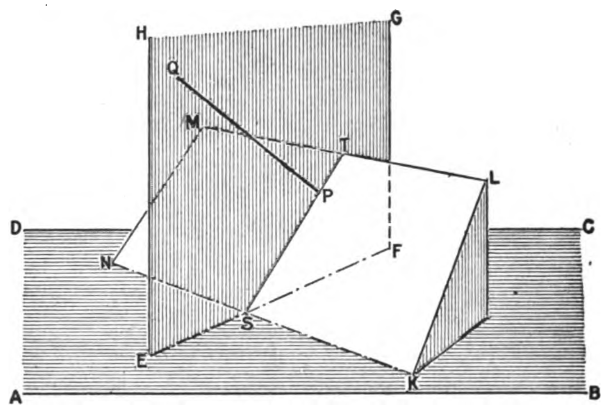


Fig. 10.

The vertical plane which contains a perpendicular to any oblique plane cuts that oblique plane in a line perpendicular to its **H.T.**, and the **H.T.**s of the two planes are at right angles to each other. In fig. 10 **A B C D** is the ground plane, **P Q** is a perpendicular to the oblique plane **K L M N** from **P**. **E F G H** is a vertical plane containing **P Q**, it intersects **K L M N** in **S T** which *must* pass through **P** and be perpendicular to **K N**. **E F**, the **H.T.** of **E F G H**, is at right angles to **K N** and passes through **S**.

VANISHING POINTS OF LINES.

The perspective representation of the vanishing point of a straight line, whether horizontal or inclined, is the intersection with the **P.P.** of a line drawn through the eye parallel to the line whose **V.P.** is required.

The line drawn through the eye is called the *Vanishing Parallel* of the original line.

In elementary perspective points and lines are placed in perspective by using horizontal and vertical planes, and the vanishing points obtained lie on the horizontal line.

In advanced perspective other planes can be used and the vanishing points need not be on the horizontal line.

VANISHING POINTS OF INCLINED LINES.

If a line is inclined its vanishing parallel will also be inclined, and therefore the vanishing point of the line is necessarily above or below the **H.L.** It also follows that any line parallel to the **P.P.** will have its perspective representation indicated by a line parallel to the original line.

In fig. 11 **P.P.**, **G.P.**, and **E** are shown in position. **A B** is a horizontal line, **A C** and **A D** are two *inclined* lines.

E V[1] the vanishing parallel of **A B** intersects the **P.P.** at **V**[1] on the **H.L.**, as **A B** is horizontal.

E V[2] the vanishing parallel of **A C** intersects the **P.P.** at **V**[2] *above* the **H.L.**, as **A C** is inclined upwards (away from spectator).

E V[3] the vanishing parallel of **A D** intersects the **P.P.** at **V**[3] *below* the **H.L.**, as **A D** is inclined downwards.

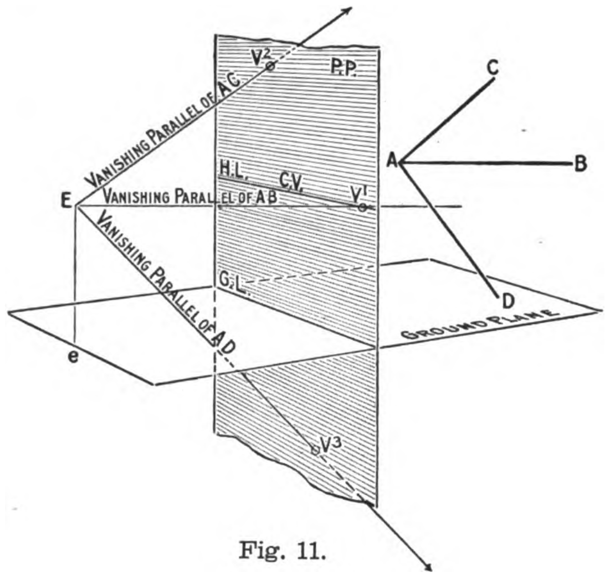


Fig. 11.

THE VARIOUS KINDS OF PLANES.

(*a*) Horizontal planes; *i.e.* planes parallel to the ground.

(*b*) Vertical planes; *i.e.* planes perpendicular to the ground.

(*c*) Ascending planes; *i.e.* planes having their **H.T.**s parallel to the **P.P.**, and *rising* as they recede from the **P.P.**

(*d*) Descending planes; *i.e.* planes having their **H.T.**s parallel to the **P.P.**, and *descending* as they recede from the **P.P.**

(*e*) Inclined planes; *i.e.* planes having their **H.T.**s perpendicular to the **P.P.** and inclined to the ground.

(*f*) Oblique planes; *i.e.* planes having their **H.T.**s neither parallel nor perpendicular to the **P.P.**, the plane being inclined to the ground.

VANISHING PLANES, VANISHING LINES OF PLANES, AND PICTURE LINES.

The Vanishing Plane of a plane is a plane through the eye parallel to the original plane.

The Vanishing Line (**V.L.**) of a plane is the line in which the plane would seem to disappear on being produced infinitely. The perspective representation of a **V.L.** is determined by obtaining the line of intersection of the plane's Vanishing plane with the **P.P.** The vanishing plane and vanishing line of a plane correspond to the vanishing parallel and vanishing point of a line.

N.B. The vanishing line of a plane contains the **V.P.** of every line on that plane.

The Centre of the Vanishing Line (**C.V.L.**) of a plane is the point of intersection with the **V.L.** of a line drawn from the eye perpendicular to the **V.L.**

The Picture Line (**P.L.**) of a plane is the line in which the plane cuts the **P.P.**

The **V.L.** of a plane, and its **P.L.** are always parallel.

N.B. The **V.L.** is infinitely distant; its perspective representation is drawn on the **P.P.**, and is generally called the **V.L.** The **P.L.** is on the **P.P.**, hence it coincides with its perspective representation.

FIGURES SHOWING THE PICTURE AND VANISHING LINES OF PLANES (figs. 12, 13, 14, 15, 16, and 17).

The figure on the right in each case corresponds to the figure on the left, and is a representation of the front view of the picture plane. The position of the eye (**E**[1]) in the front view is obtained by rotating the eye (**E**) into the picture plane.

In fig. 12. On the left:—The **P.P.**, **G.P.**, and **E** are represented in position. **A B C D** is a horizontal plane, **A D** being its **P.L.** **K L M N** is the vanishing plane of **A B C D**, giving **M L** as its **V.L.**

On the right:—The same plane is represented in perspective, the shaded lines indicating the plane from the **P.P.** till it seems to disappear on the horizon.

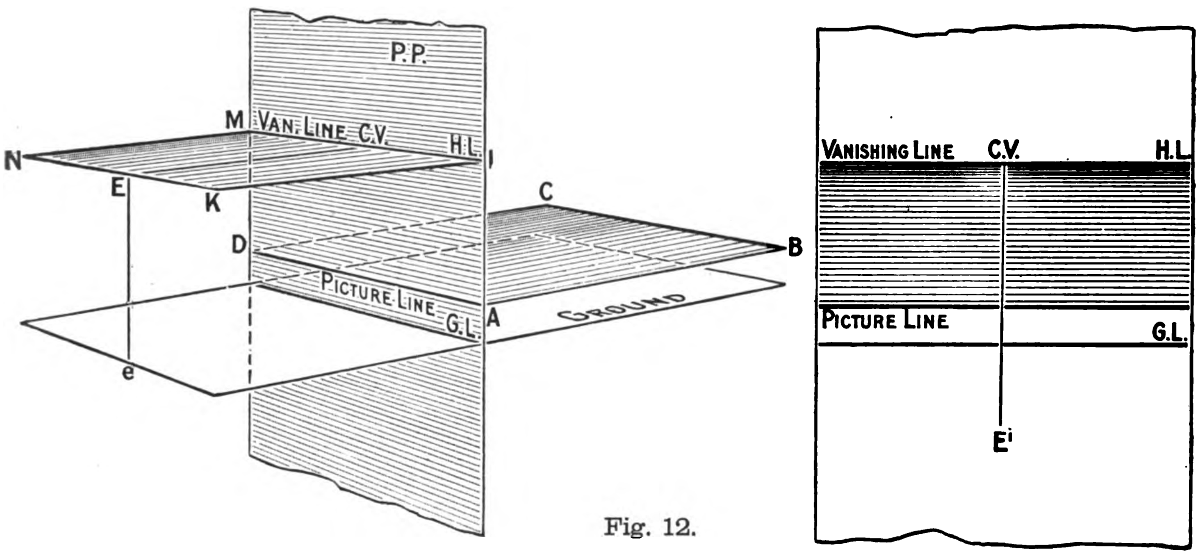


Fig. 12.

In fig. 13 **A B C D** is a vertical plane, **A D** its **P.L.** being vertical. **K L M N** is the vanishing plane of **A B C D**, giving **L M** as its **V.L.**

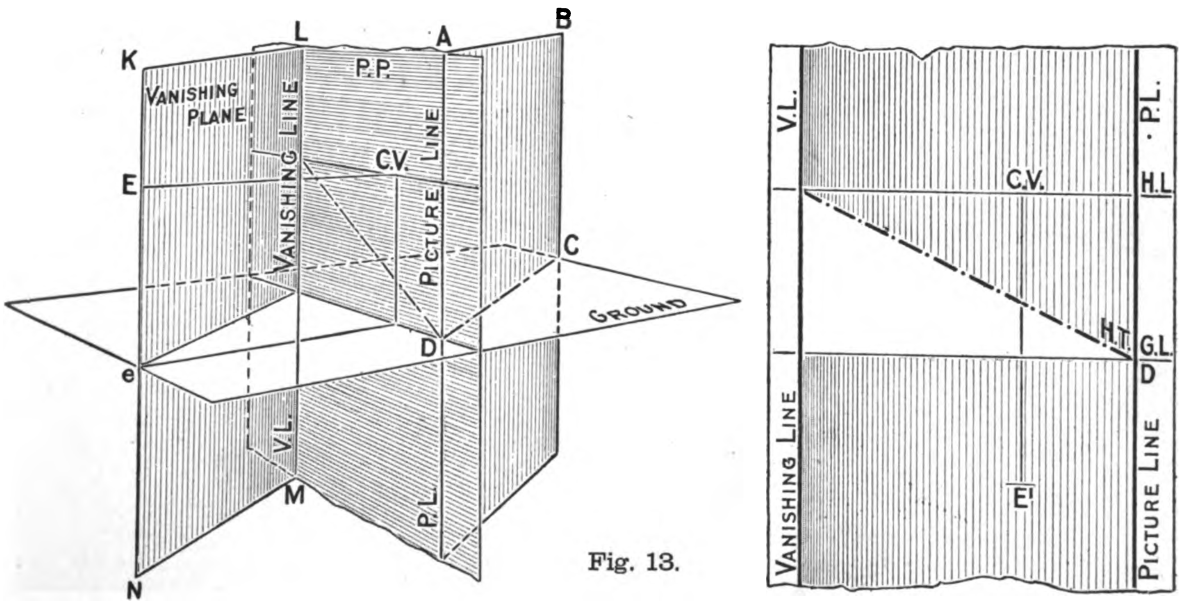


Fig. 13.

In fig. 14 **A B C D** is an ascending plane, **G H** its **P.L.** being horizontal. **K L M N** is the vanishing plane of **A B C D**, giving **L M** as its **V.L.**

In fig. 15 **A B C D** is a descending plane, **A D** its **P.L.** being horizontal. **K L M N** is the vanishing plane of **A B C D**, giving **L M** as its **V.L.**

In fig. 16 **A B C D** is an inclined plane, **A D** being its **P.L.** **K L M N** is the vanishing plane of **A B C D**, giving **L M** as its **V.L.**

In fig. 17 **A B C D** is an oblique plane, **A D** being its **P.L.** **K L M N** is the vanishing plane of **A B C D**, giving **L M** as its **V**

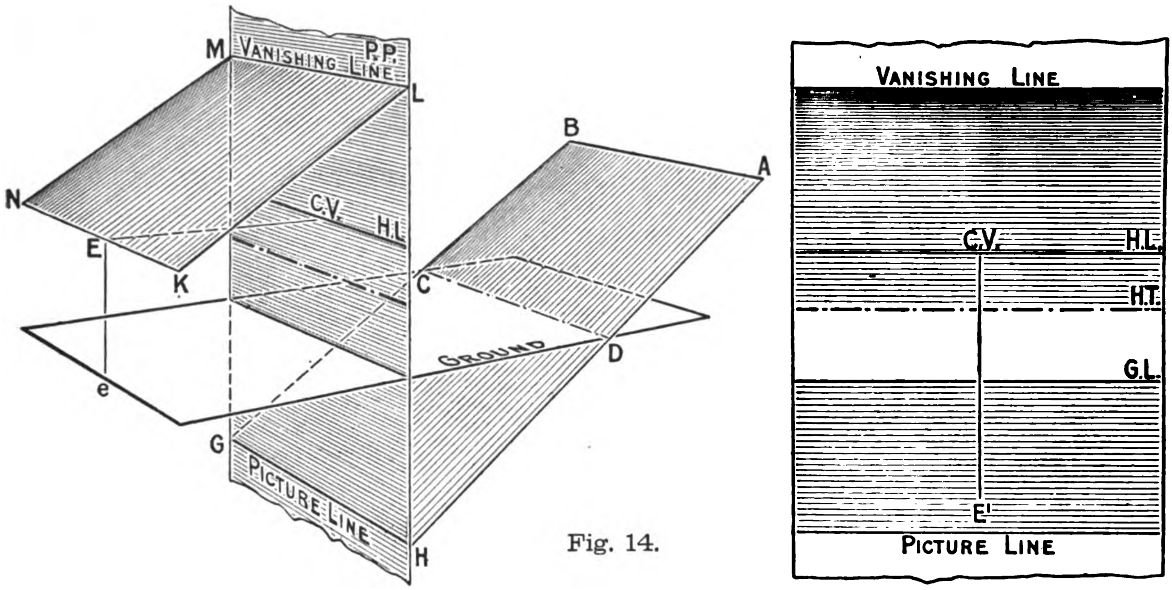


Fig. 14.

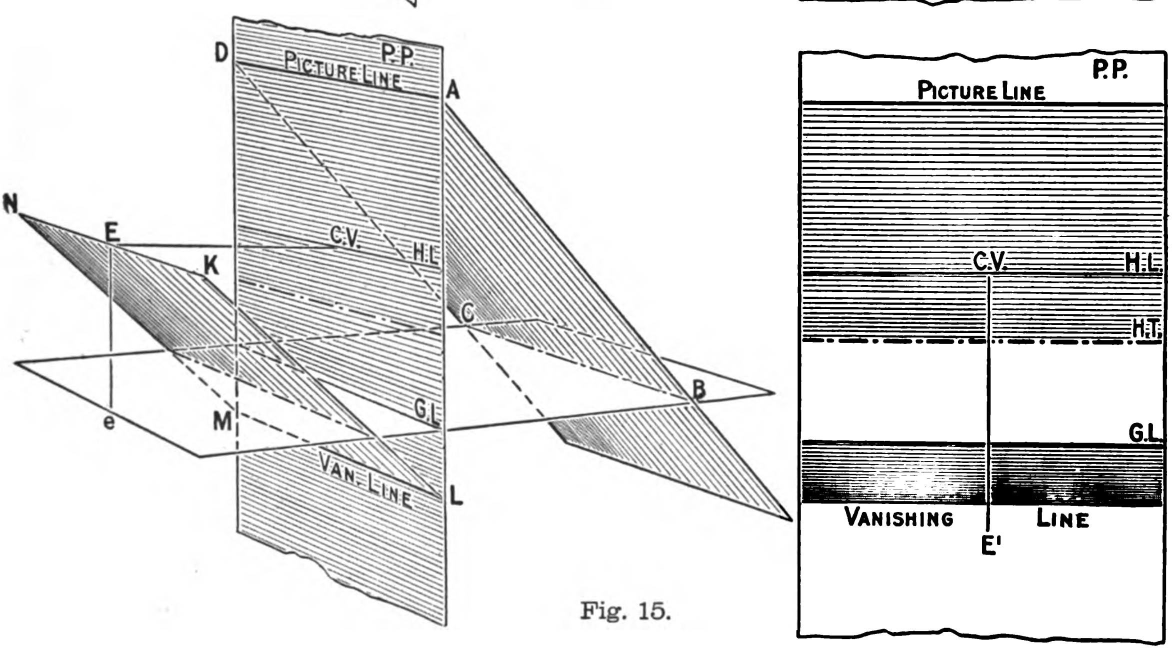


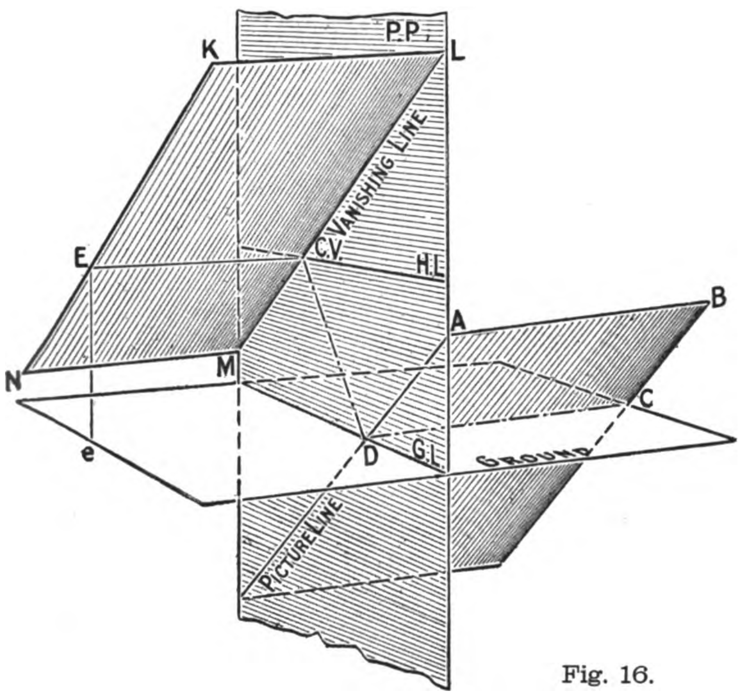
Fig. 15.

METHODS FOR OBTAINING THE PERSPECTIVE REPRESENTATION OF POINTS AND LINES BY THE USE OF THE VARIOUS PLANES.

In advanced perspective the Measuring Point Method is employed for working perspective problems, and in this method the paper upon which the drawing is made is considered to represent the picture plane.

The Measuring Point (**M.** or **M.P.**) is that **V.P.** by means of which we are able to measure given lengths on lines having that **V.P.** If a plane is assumed to contain a line the **V.P.** of the line and that **V.P.**'s corresponding **M.P.** lie on the **V.L.** of that plane.

To find the **M.P.** *of a* **V.P.**:—The **M.P.** of any **V.P.** is obtained by measuring the length of the line's vanishing parallel from the **V.P.** along the **V.L.** of a plane containing the line. In making measurements care must be taken to use the **P.L.** of the *same* plane.



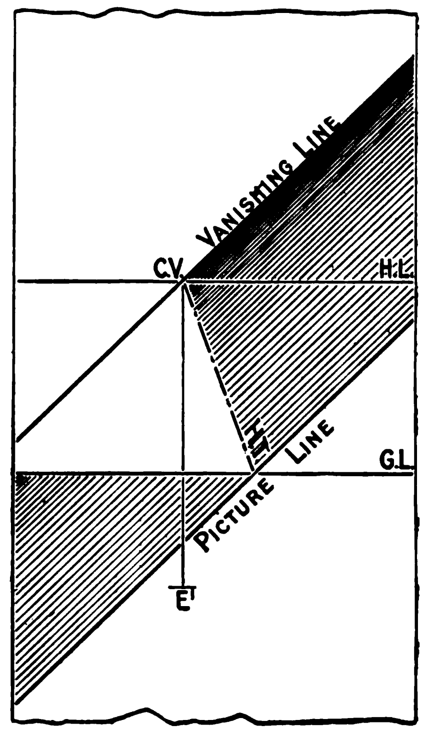
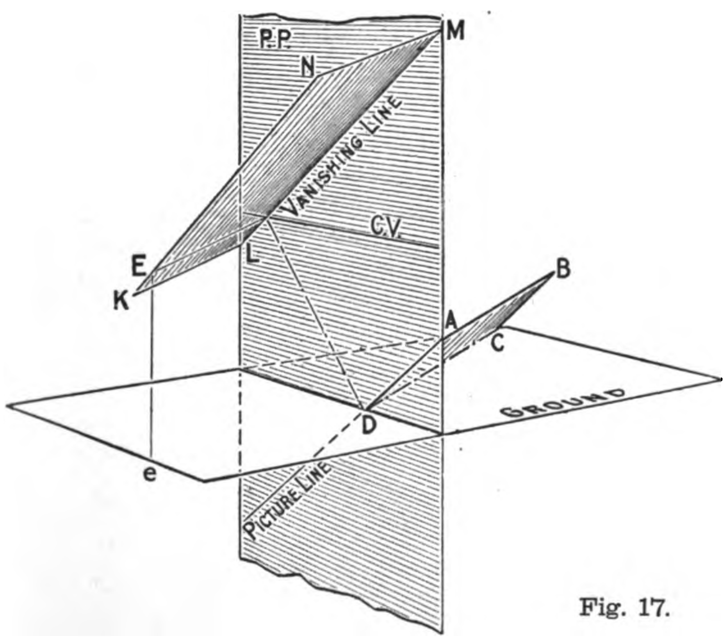


Fig. 16.



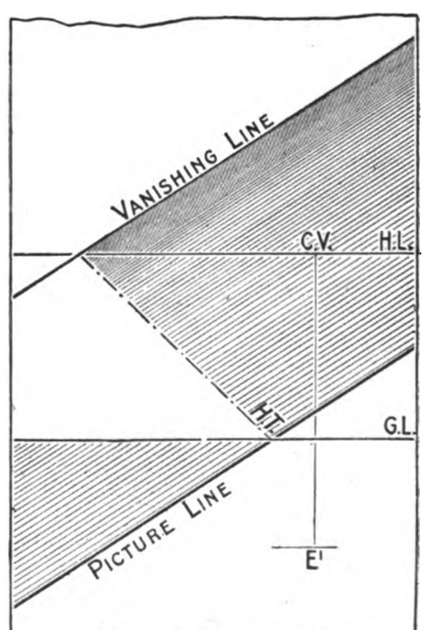


Fig. 17.

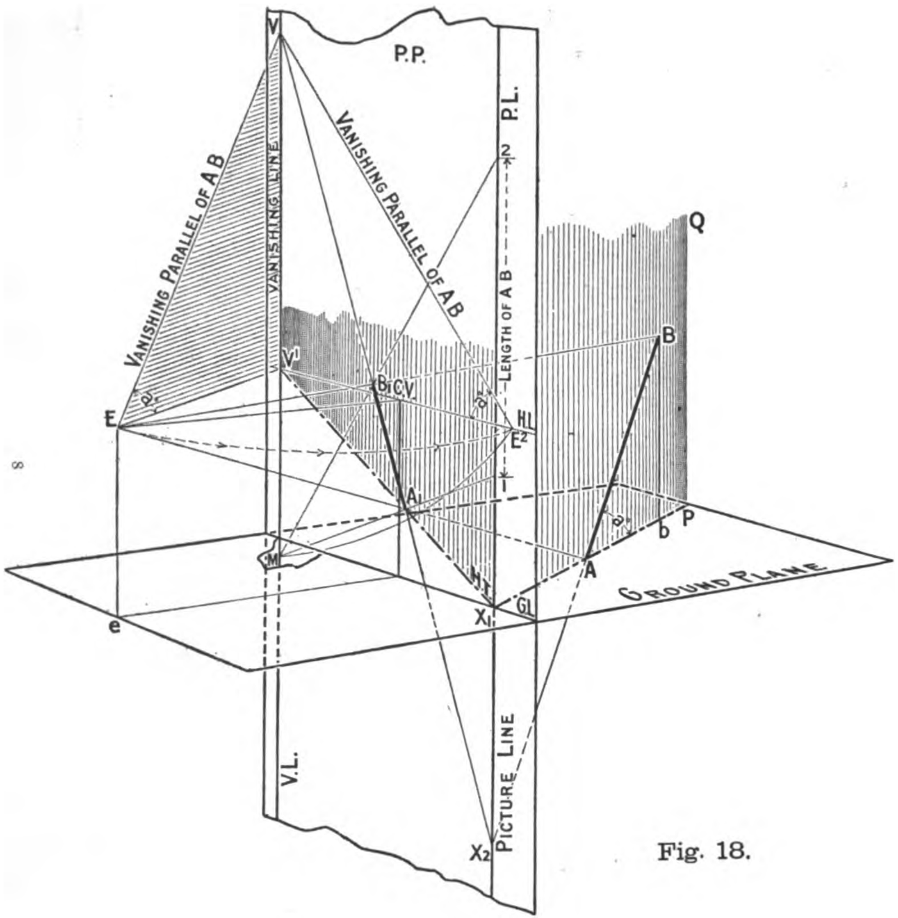


Fig. 18.

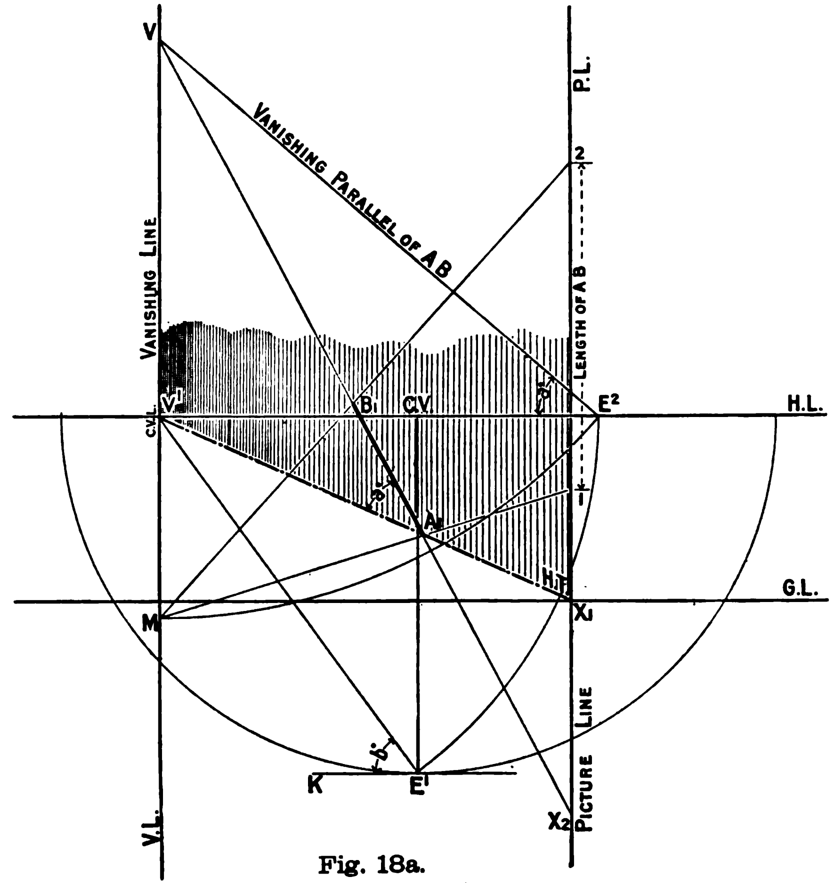


Fig. 18a.

VERTICAL PLANES.

In fig. 18 the G.P., P.P., E, and an inclined line AB are shown in position. AB has the end A on the ground, and is inclined at a° to the G.P. X_1APQ is a vertical plane containing AB. The H.T. of X_1APQ passes through A, as A is on the ground.

A vertical line through X_1 is the P.L. of X_1APQ. EV^1V is the vanishing plane of X_1APQ, giving the vertical line VV^1M as the V.L. of X_1APQ. X_1V^1 is the perspective representation of the horizontal trace X_1AP; hence the perspective representation of A will lie on X_1V^1 and is shown at A_1 on this line, AA_1E being the ray from A to the eye.

BA produced intersects the P.P. at X_2 on the P.L. of X_1APQ. EV is the vanishing parallel of AB; it intersects the P.P. at V, the vanishing point of AB. Thus X_2V is the perspective representation of AB produced infinitely from the P.P. in the direction AB. A_1 and B_1, the perspective representations of A and B, must therefore lie on X_2V, and B_1 is obtained by drawing the ray BE cutting X_2V at B_1.

The M.P. of AB is obtained (see p. 6) by measuring from V along VV^1 a length equal to VE; this, in practice, is done in the following way.

Suppose VEV^1 to be rotated into the P.P. about the line VV^1 as indicated by the arrows, and let E^2 be the position of the eye thus rotated. Since VV^1E is a right angle V^1E after rotation will lie along the H.L., and it will be evident that V^1E^2 will be equal to V^1E, *i.e.* the vanishing parallel of X_1AP, the horizontal trace of X_1APQ. VE^2 is thus equal to VE, the vanishing parallel of AB. So that the M.P. of AB can be obtained by taking V as centre and VE^2 as radius and describing an arc of a circle cutting VV^1M at the required measuring point M.

It should be observed that V^1 may be easily obtained if the angle between X_1AP and the P.P. is known, and that $\angle VE^2V^1$ is equal to $\angle VEV^1$ which is equal to $\angle BAP$, *i.e.* the inclination of AB to the G.P.

MA_1 and MB_1 produced cut the P.P. at 1 and 2 respectively. 1,2 is therefore the true length of the line A_1B_1.

Fig. 18*a* is a reproduction of the foregoing construction as it would be executed on a drawing.

The H.T. of the vertical plane which contains AB is to make an angle of b° with the P.P. towards the left. V^1 is thus obtained by making $\angle KE^1V^1$ equal to b°. E^2 is obtained by taking V^1 as centre, V^1E^1 as radius, and describing an arc of a circle cutting the H.L. at E^2. $\angle V_1E^2V$ is equal to the inclination of AB to the G.P., *i.e.* a°. M is obtained by drawing the arc E^2M with V as centre and VE^2 as radius. A_1 is taken on the H.T. MA_1 is produced to cut the P.L. at 1. 1,2 is equal to the length of AB. $M2$ cuts A_1V at B_1.

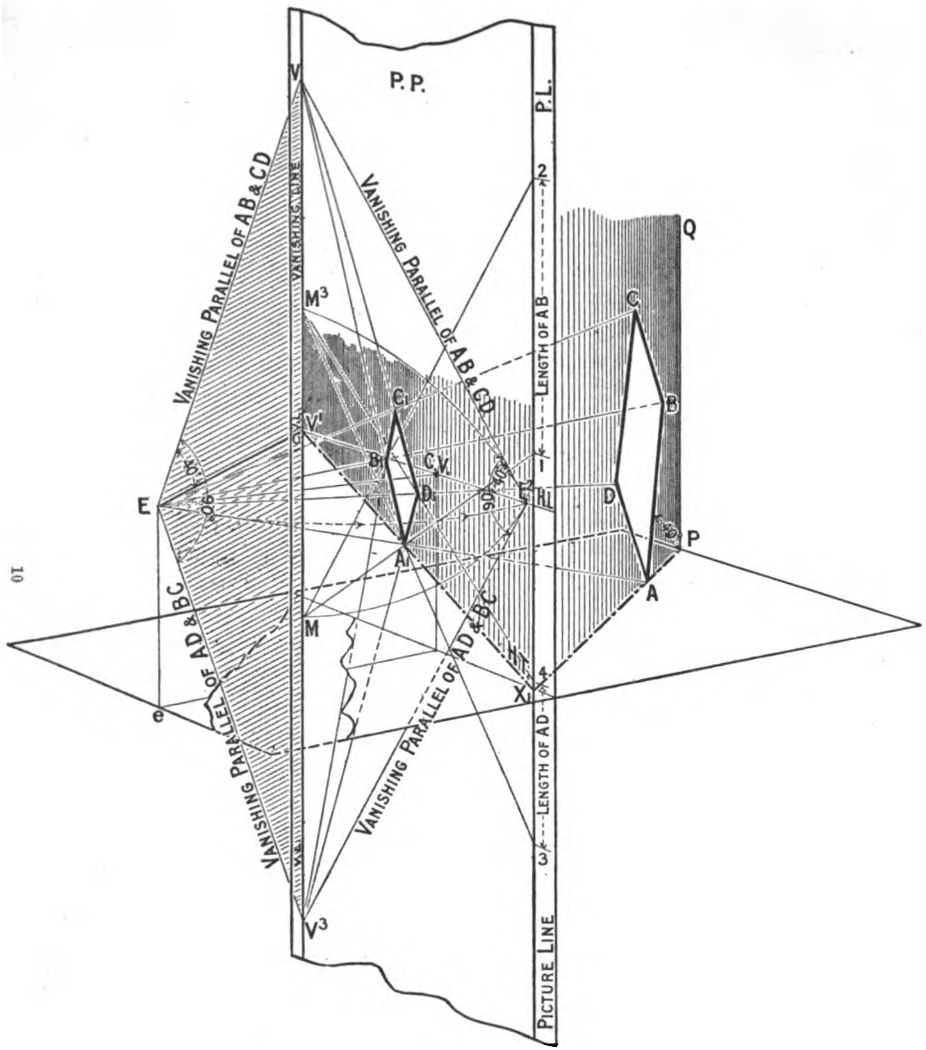


Fig. 19.

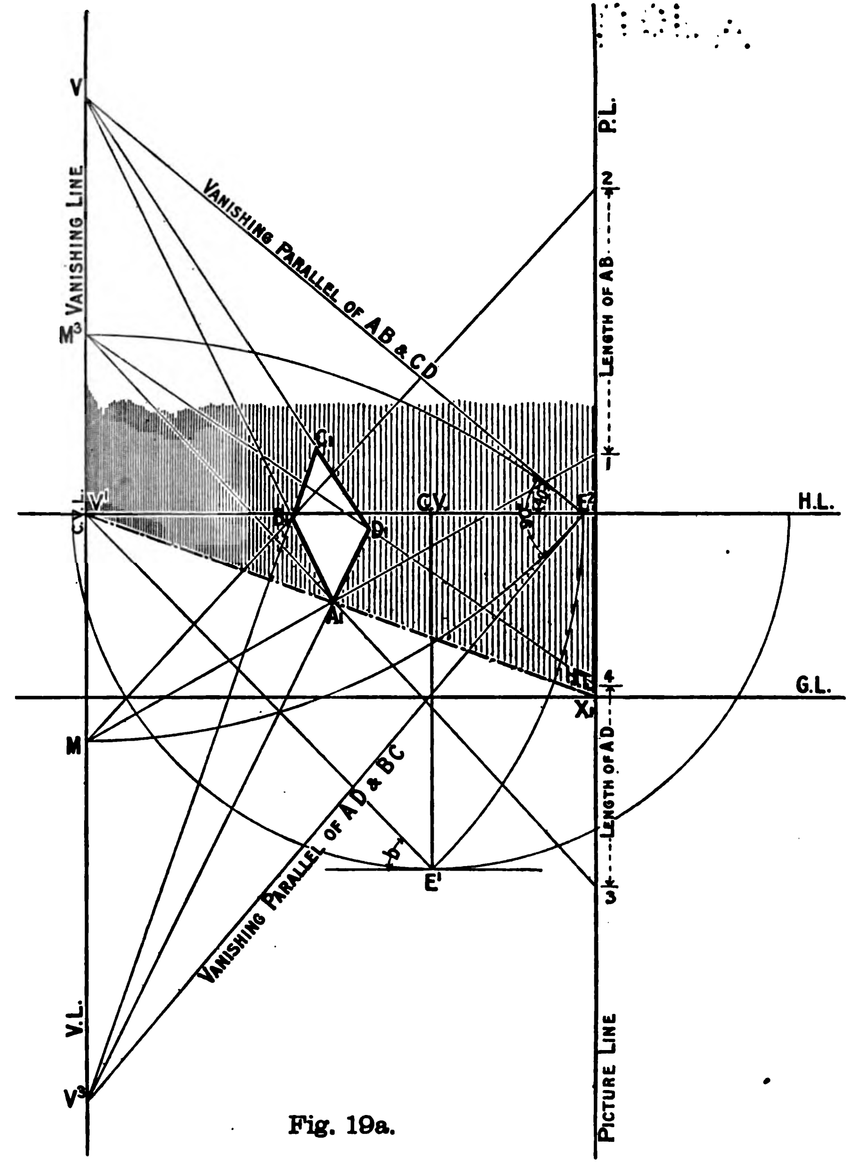
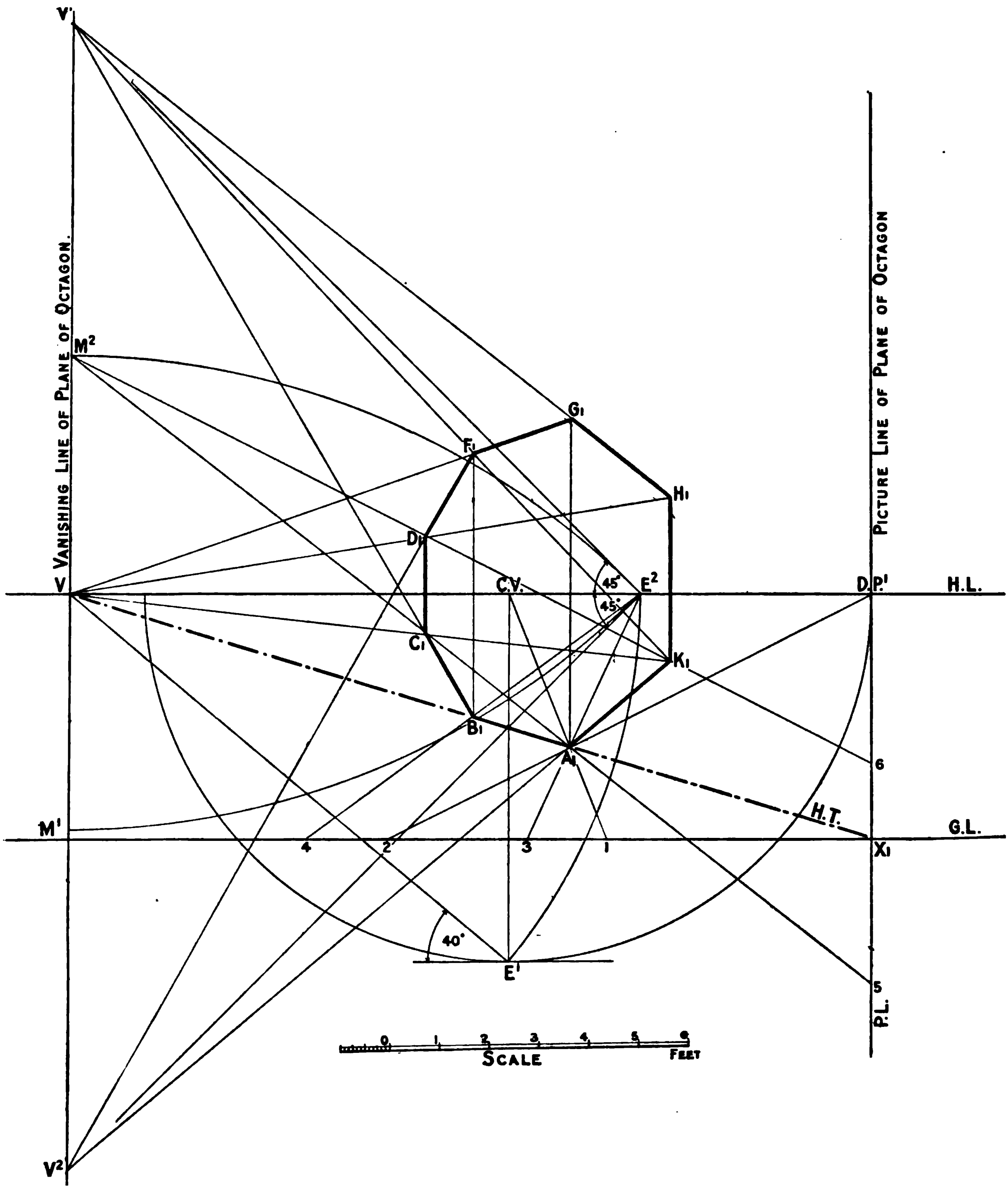


Fig. 19a.

The lettering of figs. 18 and 19 correspond. In fig. 19 $\mathbf{ABCD}$ is a rectangle with one corner ($\mathbf{A}$) on the ground, the side $\mathbf{AB}$ being inclined at 40° to the **G.P.** The plane of the rectangle is vertical, and is inclined at $\mathbf{b}°$ to the **P.P.** towards the left. $\mathbf{A_1B_1}$ has been obtained as in the figure 18. $\mathbf{EV^3}$ is the vanishing parallel of $\mathbf{DA}$. Observe that $\mathbf{VEV^3}$ is a vertical plane, and that $\mathbf{VEV^3}$ is an angle of 90°. $\mathbf{E^2}$ is the position of the eye when $\mathbf{VEV^3}$ is rotated into the **P.P.** about $\mathbf{VV^3}$, $\mathbf{VE^2V^3}$ being a right angle. The **M.P.** of $\mathbf{V^3}$ is obtained by describing an arc with $\mathbf{V^3}$ as centre and $\mathbf{V^3E^2}$ as radius, cutting $\mathbf{VV^3}$ at $\mathbf{M^3}$. $\mathbf{M^3A}$ is produced to the **P.L.** of $\mathbf{ABCD}$, and cuts it at **3**. **3,4** is equal to $\mathbf{AD}$, and cuts $\mathbf{V^3A_1D_1}$ at $\mathbf{D_1}$. Since $\mathbf{DC}$ and $\mathbf{CB}$ are parallel to $\mathbf{AB}$ and $\mathbf{DA}$ respectively, their perspective representations must vanish at $\mathbf{V}$ and $\mathbf{V^3}$.

Fig. 19*a* is a reproduction of the foregoing construction as it would be executed on a drawing.

PROBLEM I.



V'
M²
Vanishing Line of Plane of Octagon.
Picture Line of Plane of Octagon
G₁
F₁
H₁
D₁
V
C.V.
45°
E²
D.P'
H.L.
45°
C₁
K₁
B₁
A₁
6
H.T.
M'
G.L.
4
2
3
1
X₁
40°
E'
5
P.L.
0
1
2
3
4
5
6
Scale
Feet
V²

PROBLEMS INVOLVING THE USE OF VERTICAL PLANES.

PROBLEM I.

An octagon of 4 *ft.* 6 *ins. side stands vertically with one edge on the ground. The nearest corner of the figure on the ground to the* **P.P.** *is* 2 *ft. on the spectator's right, and* 4 *ft.* 6 *ins. from the ground line. Show the perspective representation of the figure when its plane makes an angle of* 40° *with the* **P.P.** *towards the left, the height of the eye above the ground to be taken as* 5 *ft., and the distance of the eye from the* **P.P.** *as* 7 *ft.* 6 *ins. Scale* ½ *in. to* 1 *ft.*

A_1 is on the **G.P.**, 2 ft. on the right, and 4 ft. 6 ins. beyond the **P.P.**

A B, the edge on the ground, will vanish on the **H.L.** at **V**.

$V^1V\,V^2$, a vertical line through **V**, is the vanishing line of the plane of the octagon.

Since **A B** is on the ground, $V\,B_1A_1$ (produced) is the **H.T.** of the plane containing the octagon, and this line cuts the **P.P.** at X_1 on the **G.L.** The picture line of the plane of the octagon is a vertical line through X_1.

The edge **A K** of the octagon is inclined at 45° with the ground (notice that this line goes downward away from the **P.P.**). E^2V^2 is its vanishing parallel when rotated into the **P.P.**, giving V^2 as its **V.P.**

B C is inclined at 45° with the ground (notice that this line is inclined upwards away from the **P.P.**). E^2V^2 is its vanishing parallel when rotated into the **P.P.**, giving V^1 as its **V.P.**

M^2 is the **M.P.** of V^2.

E^2 is the **M.P.** of **V**.

Make A_1B_1 4 ft. 6 ins. long (use E^2 and the **G.L.**).

Produce V^2A_1, join M^2A_1 and produce to cut the **P.L.**, at **5** measure off **5,6**, 4 ft. 6 ins. long; join **6** M^2, cutting V^2A_1 produced at K_1.

K C is horizontal, hence K_1C_1 vanishes at **V**. B_1C_1 vanishes at V^1, and cuts K_1V at C_1.

C_1D_1 and K_1H_1 are vertical.

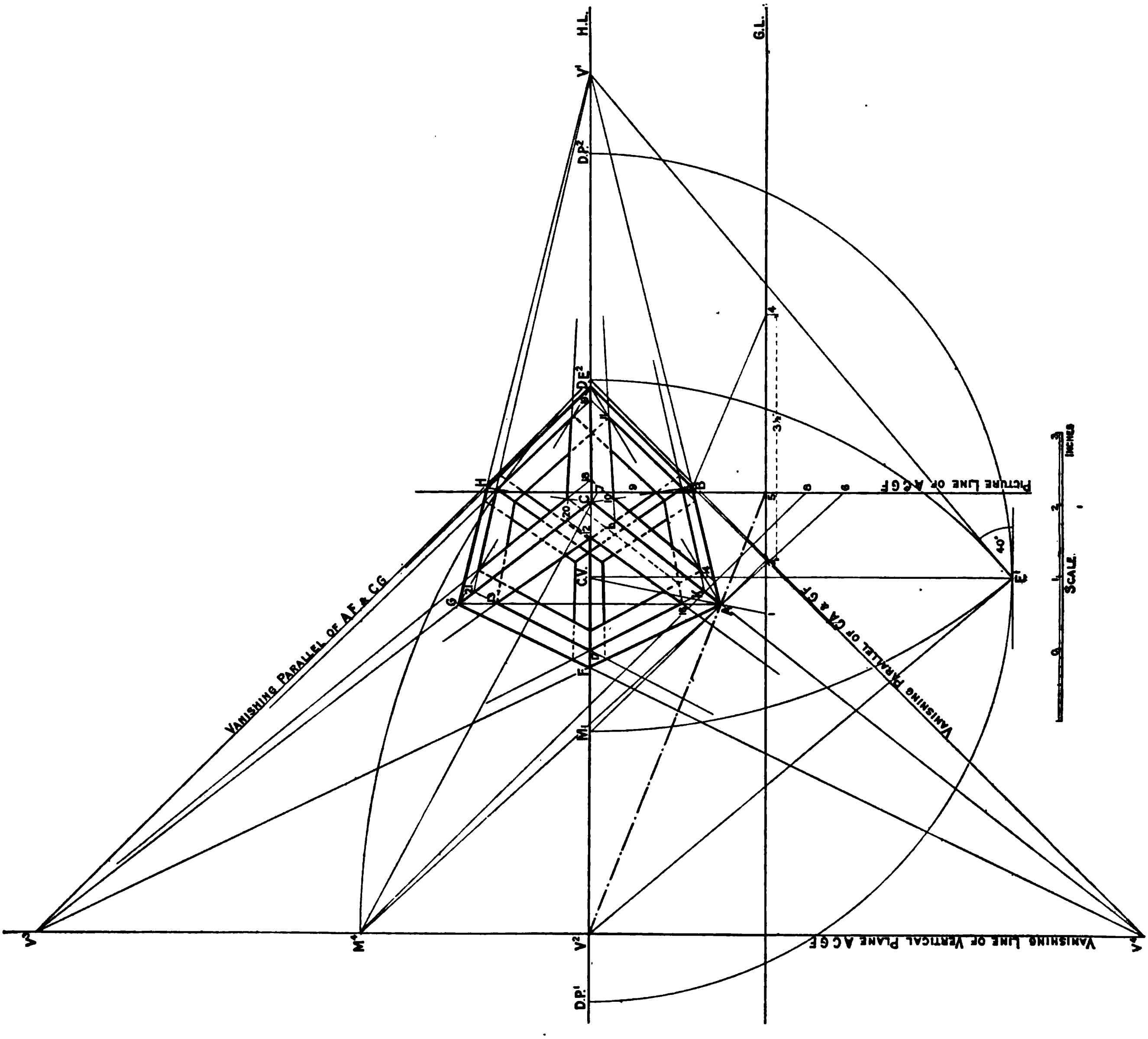
K F is parallel to **B C**, therefore K_1F_1 vanishes at V^1.

D F is parallel to **A K**, therefore D_1F_1 vanishes at V^2.

F_1G_1 vanishes at **V**, and finally, as **G H** is parallel to **B C**, G_1H_1 vanishes at V^1.

$A_1B_1C_1D_1F_1G_1H_1$ is the required representation.

PROBLEM II.


H.L.
G.L.
Vanishing Parallel of AF & CG
Picture Line of A C G F
Vanishing Parallel of CA & GF
Vanishing Line of Vertical Plane A C G E
Scale
Inches

PROBLEM II.

Give a perspective representation of a skeleton cube, with one edge on a horizontal plane 2½ ins. below the eye, and the undermost face inclined upwards to the left at an angle of 45° to the horizontal plane.

Let the edge upon which the cube rests start at a point ½ in. to the left of the centre and 2 ins. within the picture, and draw it towards the right at an angle of 40° to the picture plane; measure it 3½ ins. long, and complete the cube in the above position. Make the distance of the eye 6 ins.

Obtain the **H.L.** and **G.L.** 2½ ins. apart, as one edge is 2½ ins. below the eye.

Find **A B** the perspective representation of the edge of the cube which is on the ground.

Notice that if a cube has one edge on the ground, two faces of the cube must be vertical. (**A C G F** is one of these faces.)

Suppose a vertical plane to contain **A C G F**, its **H.T.** will pass through **A** and be at right angles to **A B**, hence E^1V^2 is at right angles to E^1V^1. $V^3V^2V^4$ is the vanishing line of the vertical plane **A C G F**. V^2A produced cuts the **G.L.** at **5**. A vertical line through **5** determines the **P.L.** of **A C G F**.

The face **F A B** of the cube is inclined at 45° to the horizontal plane, hence **A F** makes 45° with the **H.T.** of **A C G F**, *i.e.* V^2A.

Rotating the vanishing parallel of **A F** into the **P.P.**, $V^2E^2V^3$ is an angle of 45°.

C A is at right angles to **A F**; its vanishing parallel must be at right angles to the vanishing parallel of **C A**, and thus its **V.P.** will be at V^4, $V^3E^2V^4$ being 90°.

Obtain M^4 the **M.P.** of V^4.

Join M^4A and produce to cut the **P.P.** at **6** (on the **P.L.** of **A C G F**). Measure off on the **P.L.** **6,7** equal to the side of the cube (3½ ins.).

Join **7** to M^4, cutting V^4A produced at **C**.

A F and **C G** vanish at V^3.

As **A F** and **A C** are inclined at 45° to the **G.P.**, therefore **A G** is vertical, hence obtain **G** in CV^3 by drawing **A G** vertically through **A**; join GV^4, cutting AV^3 at **F**. This completes the face **A C G F**.

C D and **G H** are parallel to **A B**, and therefore vanish at V^1 on the **H.L.** **B D** and **D H** vanish at V^4 and V^3 respectively. This completes the drawing of the cube.

Draw the diagonals on each visible face.

Obtain **K** and **L** on **A C** by making **6,8** and **7,9** each ½ in. long, and joining **8** and **9** to M^4.

Join **L** to V^1 and V^3, and **K** to V^1 and V^3; these lines determine **10, 11, 12, 13, 14, 15, 16,** and **17** on the diagonals of the faces. **10,14** and **11,15** should vanish at V^4, and these lines produced cut **C D** at **18** and **19** respectively. **12,16** and **13,17** should vanish at V^4, and these lines cut **C G** at **20** and **21** respectively.

Join **18** and **19** to V^3, and **20** and **21** to V^1.

A square has now been completed on each of the three visible faces. Complete the skeleton cube as shown.

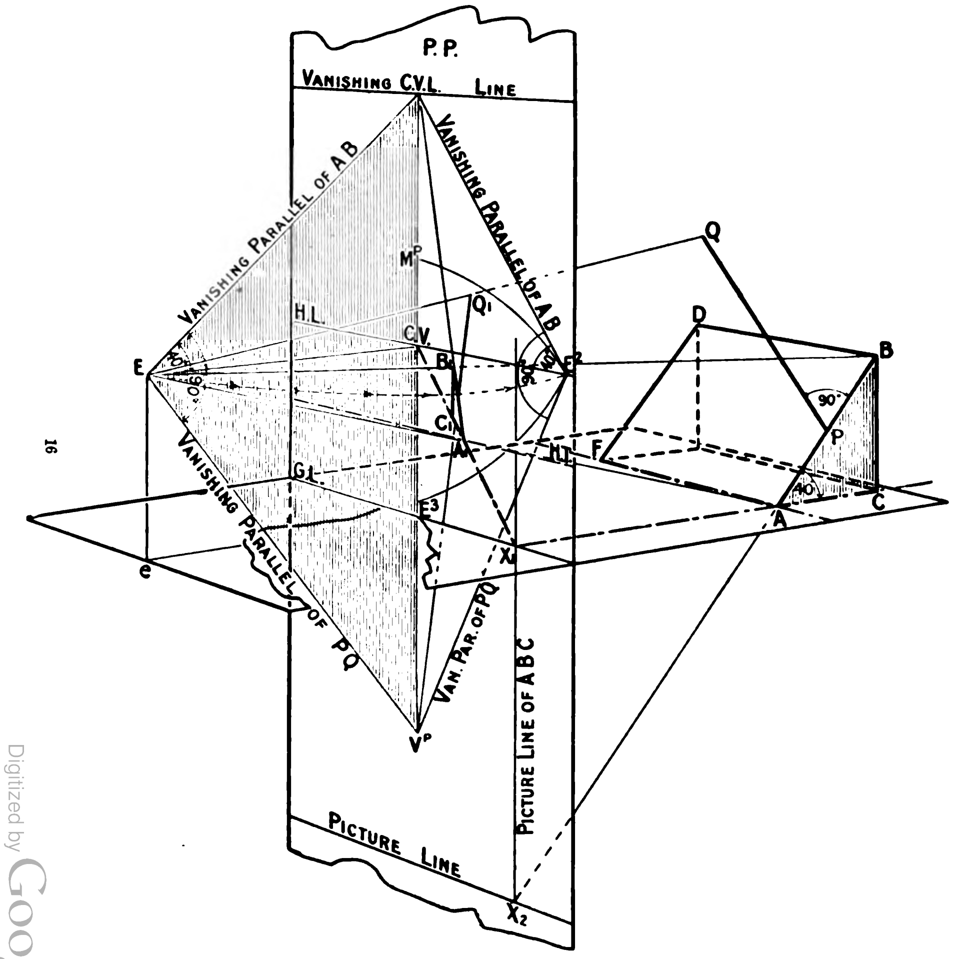


Fig. 20.

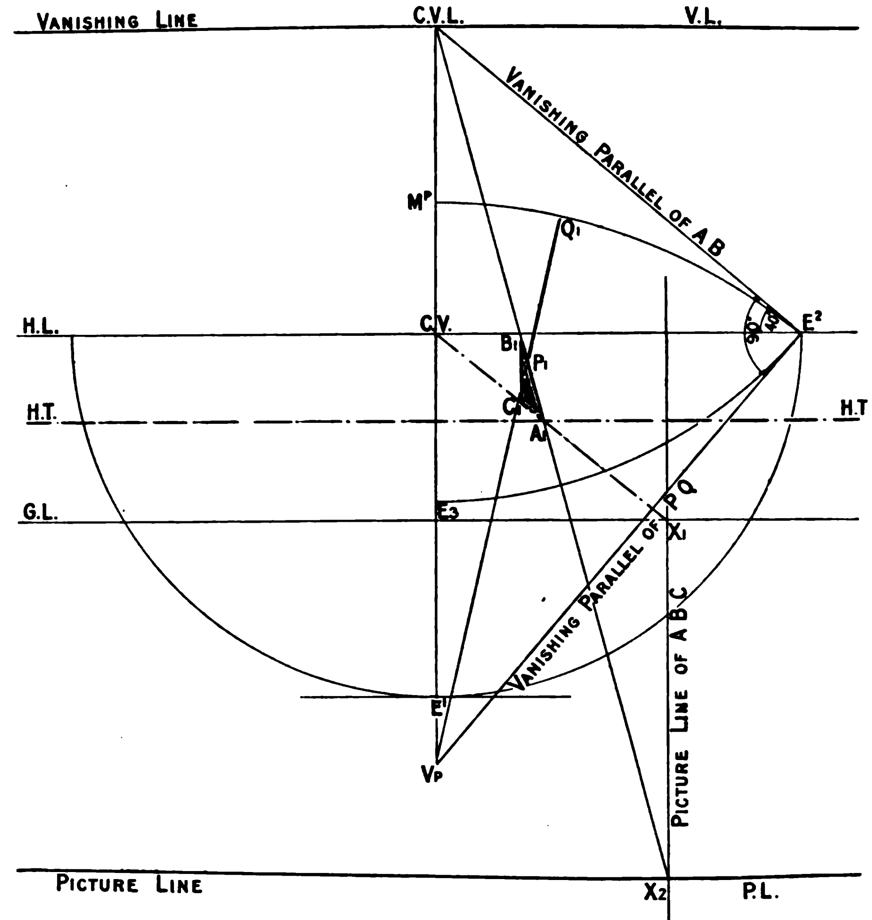


Fig. 20a.

ASCENDING PLANES.

Fig. 20 shows the eye, **P.P.**, and an ascending plane **A B D F** in position. **A B D F** is inclined at 40° to the ground, and **A F** is its horizontal trace. **P Q** is a perpendicular to **A B D F** from the point **P** which is on the plane.

Suppose **A B D F** to be cut by a vertical plane **B A C** containing **P**, so that the horizontal trace **A C** of **B A C** is at right angles to **A F**. **A B** is the intersection of **B A C** with **A B D F**.

Observe that **A C** will be at right angles to the **P.P.**, and that **B A F** will be a right angle.

To determine the **V.L.** *of* **A B D F**:—Through **E** draw **E C.V.L.** parallel to **A B**, cutting the **P.P.** at **C.V.L.**; since **A B C** is a vertical plane at right angles to the picture plane **C.V.L.**, **C.V.** will be vertical. The angle **C.V. E C.V.L.** will be equal to ∠ **B A C**, *i.e.* the inclination of the plane to the ground.

C.V.L. is the **V.P.** of **A B**; the **V.L.** of **A B D F** is obtained by drawing a horizontal line through **C.V.L.**

To determine the **P.L.** *of* **A B D F**:—The vertical plane **B A C** cuts the ground along **C A** and the **P.P.** in a vertical line X_1X_2 through X_1 on the **G.L.** Since **B A** lies in **B A C** it will cut the **P.P.** at X_2, where **B A** produced cuts X_1X_2.

However, as **B A** also lies on **A B D F**, the **P.L.** of **A B D F** must pass through X_2, and it is horizontal.

To determine the **V.P.** *of the perpendicular to* **A B D F**:—The vertical plane **B A C** contains **P Q**; **C.V.L. C.V.** V^P is the vanishing line of this plane. **E** V^P is parallel to **Q P**, and cuts the **P.P.** at V^P.

V^P is the vanishing point of the lines perpendicular to **A B D F**.

Observe that **C.V.L. E** V^P is a right angle.

Suppose **C.V.L. E** V^P to be rotated into the **P.P.** about **C.V.L.**V^P. **C.V.L.** and V^P would remain fixed, but **E** would be on the **P.P.** at E^2. **C.V.** E^2 being equal to **C.V. E**, the angle **C.V. E C.V.L.** would be equal to the inclination of **A B D F** to the **G.P.** **C.V.L.** E^2V^P would be a right angle.

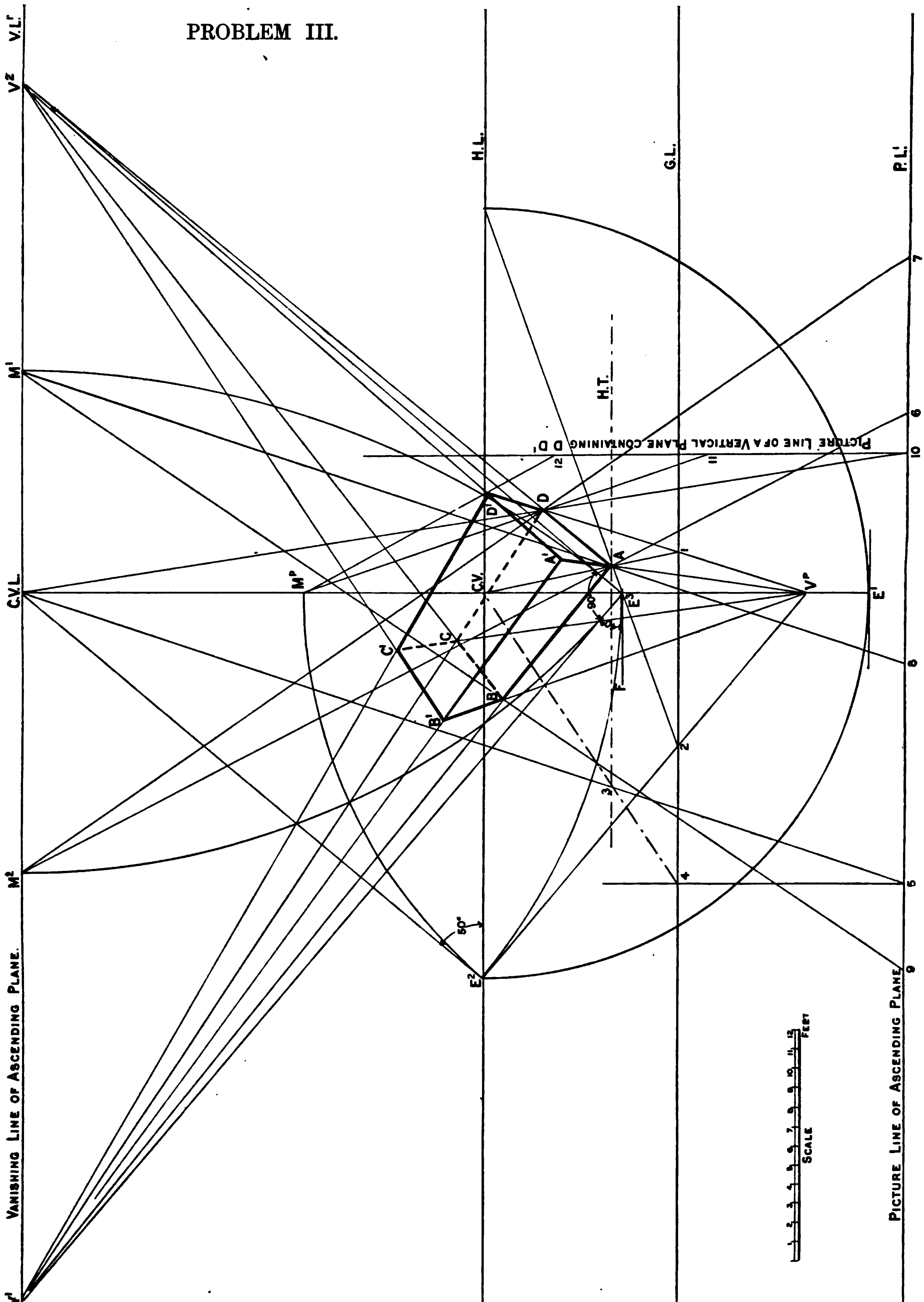
The **M.P.** of V^P, *i.e.* M^P, may be obtained by describing an arc with V^P as centre and V^PE^2 as radius.

If the vanishing plane of **A B D F** was rotated into the **P.P.** about the vanishing line, the eye would be at E^3. **C.V.L.** E^3 being equal to **C.V.L. E**, which is also equal to **C.V.L.** E^2.

Fig. 20*a* is a reproduction of the foregoing construction as it would be executed on a drawing.

The horizontal trace of the plane is a horizontal line passing through **A**.

PROBLEM III.



V.L.'
V²
M¹
C.V.L.
M²
V¹
VANISHING LINE OF ASCENDING PLANE.
H.L.
G.L.
P.L.'
H.T.
PICTURE LINE OF A VERTICAL PLANE CONTAINING D D'
C.V.
Mᴾ
Vᴾ
E¹
E²
E³
50°
90°
SCALE
FEET
PICTURE LINE OF ASCENDING PLANE

PROBLEMS INVOLVING THE USE OF ASCENDING PLANES.

PROBLEM III.

A square prism (side of square 8 ft., axis of prism 16 ft.) rests with one corner (**A**) *on the ground 2 ft. on spectator's right and 10 ft. from the* **P.P.** *It rests on a plane which is inclined at 50° to the ground, the horizontal trace of the plane being parallel to the* **P.P.** *One of the long edges of the prism makes an angle of 40° with the* **H.T.** *of the plane, upon which it rests, towards the left. Show the perspective representation of the prism when the eye is 10 ft. above the* **G.P.** *and 20 ft. distant from the* **P.P.** *Scale ¼ in. to 1 ft.*

Find the point **A**. The chain line **3 A** is the **H.T.** of the ascending plane containing the lower face of the prism.

With **C.V.** as centre and radius **C.V.** E^1, find E^2 on the **H.L.** Set off the angle **C.V.** E^2 **C.V.L.** equal to the plane's inclination, *i.e.* 50°. Through **C.V.** draw a vertical line to cut E^2 **C.V.L.** at **C.V.L.** A horizontal line through **C.V.L.** is the **V.L.** of the plane of **A B C D**.

Suppose a vertical plane perpendicular to the **P.P.** to contain any point **3** on the **H.T.** of **A B C D**; the chain line **C.V. 3,4** is the intersection of that vertical plane with the ground. **C.V.L.3** is the intersection of the same plane with **A B C D**. This vertical plane cuts the **P.P.** in a vertical line **4,5** through **4**. **C.V.L. 3** then cuts the **P.P.** at **5** in this line. The **P.L.** of the ascending plane is a horizontal line through **5**.

(In finding this **P.L.**, the point **A** could have been used in a similar way to point **3**, but as the lines of the construction would be almost parallel to each other, their intersections could not be accurately determined, thus it is advisable to assume a point such as at **3**.)

Find V^P the vanishing point of the perpendiculars, and its **M.P.** Consider the eye with the vanishing parallels of **A B** and **A D** to be rotated into the **P.P.** about V^1V^2; the eye would lie at E^3 on **C.V.L.** V^P. **C.V.L.** E^3 being equal to **C.V.L.** E^2. If a horizontal line E^3F is drawn through E^3 it would represent the vanishing parallel of the horizontal trace of **A B C D** (rotated into the **P.P.**). $F E^3V^1$ would be = 40°, and $V^1E^3V^2$ would be a right angle.

Find M^1 the **M.P.** of V^1 by making V^1M^1 = the vanishing parallel of **A B**, which is the same length as V^1E^3. Similarly find M^2 the **M.P.** of V^2.

Join **A** V^2 and **A** V^1. Produce M^2**A** to cut **P.L.**^{1} at **6**, measure off **6,7** 8 ft. long, and join **7** M^2 cutting **A** V^2 at **D**.

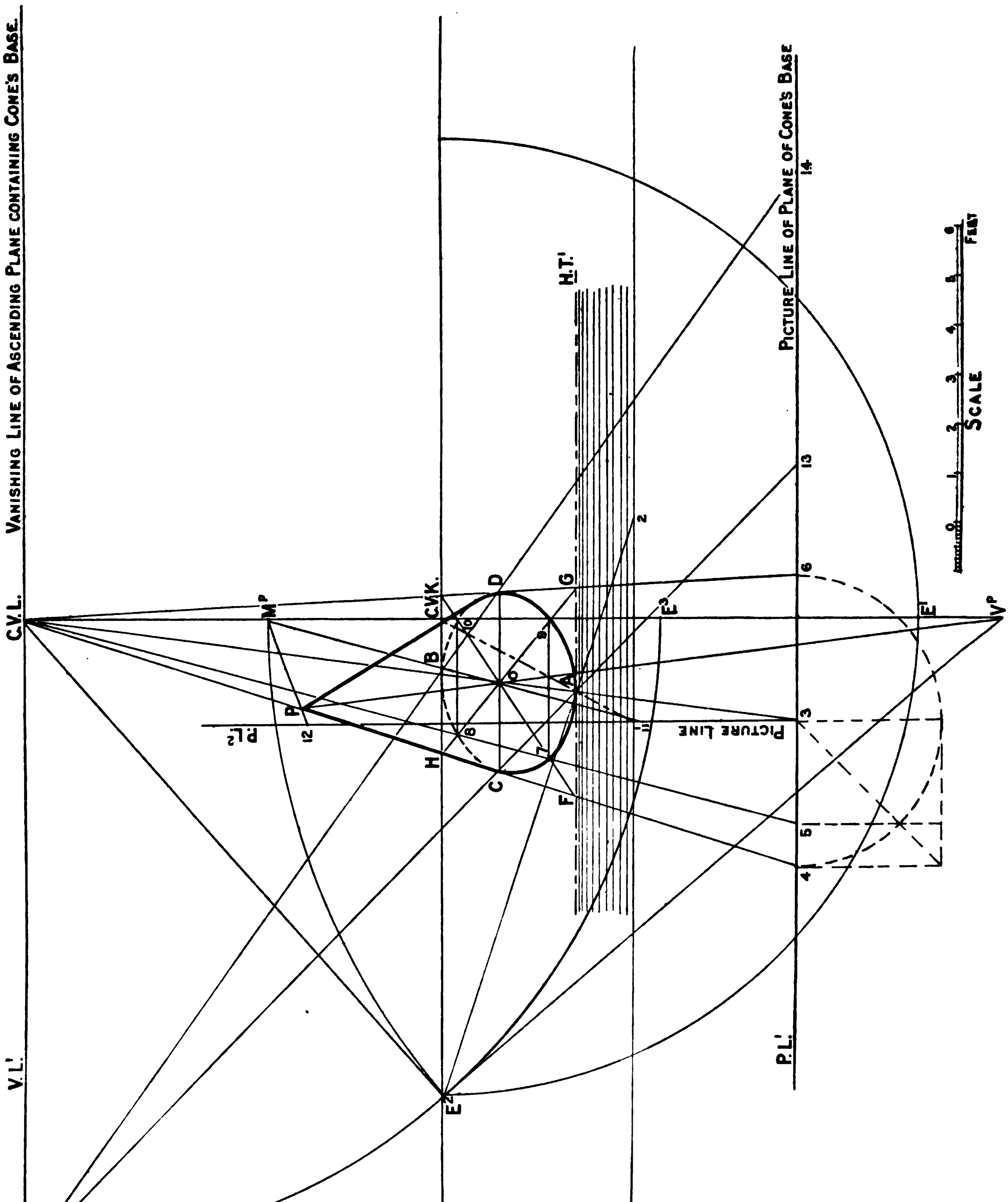
Produce M^1**A** to cut **P.L.**^{1} at **8**, measure off **8,9** 16 ft. long, and join **9** M^1, cutting **A** V^1 at **B**. **B C** and **D C** vanish at V^2 and V^1 respectively. This completes one of the largest faces.

Draw the perpendiculars **A** A^1, **B** B^1, **C** C^1, and **D** D^1, these lines vanish at V^P.

To measure the thickness of the prism on **D** D^1:—Suppose a vertical plane at right angles to the **P.P.** to contain **D** D^1, this plane cuts the ascending plane along **C.V.L. D**. **C.V.L. D** cuts the **P.P.** at **10** on **P.L.** (as **C.V.L. D** lies on the ascending plane **A B C D**), hence the assumed vertical plane cuts the **P.P.** in a vertical line through **10**. This Picture Line may be used for measuring lengths on **D** D^1, but observe that it would be a different plane which would contain any of the other perpendiculars, and hence these planes would have different picture lines.

Produce M^P**D** to cut **P.L.** of the vertical plane containing **D** D^1 at **11**, measure off **11,12** on this **P.L.** equal to the length of **D** D^1, *i.e.* 8 ft.; join **12** M^P, cutting **D** D^1 at D^1. Produce V^2D^1 to cut **A** A^1 at A^1; join A^1V^1, cutting **B** B^1 at B^1, and finally join B^1 and D^1 to V^2 and V^1 respectively. **B C**, **C D**, and **C** C^1 are not seen.

PROBLEM IV.



Vanishing Line of Ascending Plane containing Cone's Base.
C.V.L.
V.L.'
M'
M^P
E²
E³
E¹
V^P
C.V.K.
H.T.'
P.L.²
P.L.'
Picture Line
Picture Line of Plane of Cone's Base
Scale
Feet

PROBLEM IV.

Draw the perspective representation of a right cone (radius of base 3 ft., axis 7 ft.) when its base rests on a plane whose horizontal trace is parallel to the ground line, the plane being inclined at 40° to the ground. The point of contact of the base with the ground is 2 ft. on the spectator's left and 4 ft. within the picture. The height of the eye above the **G.P.** *to be taken as 4 ft. and its distance from the* **P.P.** *10 ft. Scale ½ in. to 1 ft.*

Find **A** on the ground 2 ft. on the left and 4 ft. from the **P.P.** **A** is the point of contact of the cone's base with the ground. **F A G**, a horizontal line through **A**, is the **H.T.** of the plane of the cone's base (**H.T.**^{1}).

Consider a vertical plane perpendicular to the **P.P.** to contain **A**. The chain line **C.V. A** is the intersection of this plane with the **G.P.** **C.V.L. A** is its intersection with the ascending plane. **C.V. A** cuts the **P.P.** at **1** on the **G.L.** This imaginary plane cuts the **P.P.** in a vertical line **1,3** through **1**. **C.V.L. A** therefore cuts the **P.P.** at **3** on **1,3**. The picture line of the ascending plane is a horizontal line through **3** (**P.L.**^{1}).

Circumscribe the circle with a square having one side on **H.T.**^{1}; two sides of this square are parallel to the **P.P.** and the other two will vanish at **C.V.L.** A semicircle is indicated on **4,6** for determining the points **4**, **5**, and **6**. Join these points to **C.V.L.** and let **4 C.V.L.** and **6 C.V.L.** cut **H.T.**^{1} at **F** and **G**.

Find **E**^{3}. **M**^{1}, the **M.P.** of lines vanishing at **C.V.L.**, is obtained by drawing an arc of a circle with **C.V.L.** as centre and **C.V.L. E**^{3} as radius, cutting **V.L.**^{1} at **M**^{1}.

Produce **M**^{1}**A** to cut **P.L.**^{1} at **13**, measure off **13,14** equal to **4,6**. Join **14 M**^{1} cutting **A C.V.L.** at **B**.

Through **B** draw a horizontal line **H K** cutting **F C.V.L.** and **G C.V.L.** at **H** and **K** respectively.

F G K H is the square surrounding the circle.

Draw the diagonals **F K** and **G H**; they intersect at **O** the centre of the circle. Through **O** draw a horizontal line **C O D** cutting **F H** and **G K** at **C** and **D** respectively. **A**, **D**, **B**, and **C** are four points on the circle.

5 C.V.L. cuts **F K** and **H G** at **7** and **8** respectively. Horizontal lines through **7** and **8** cut **G H** and **K F** at **9** and **10** respectively.

Draw an ellipse through **A**, **9**, **D**, **10**, **B**, **8**, **C**, and **7**.

To find the vertex:—The vertex lies in the perpendicular to the base through **O**, therefore join **V**^{P}**O** and produce it.

The vertical plane whose picture line is **P.L.**^{2} contains **O P**, therefore produce **M**^{P}**O** to cut **P.L.**^{2} at **11**.

On **P.L.**^{2} measure off **11**, **12**, the length of the cone's axis, *i.e.* 7 ft.; join **12 M**^{P}, cutting **O P** at **P**.

P is the vertex.

Complete the representation by drawing tangents to the base from **P**.

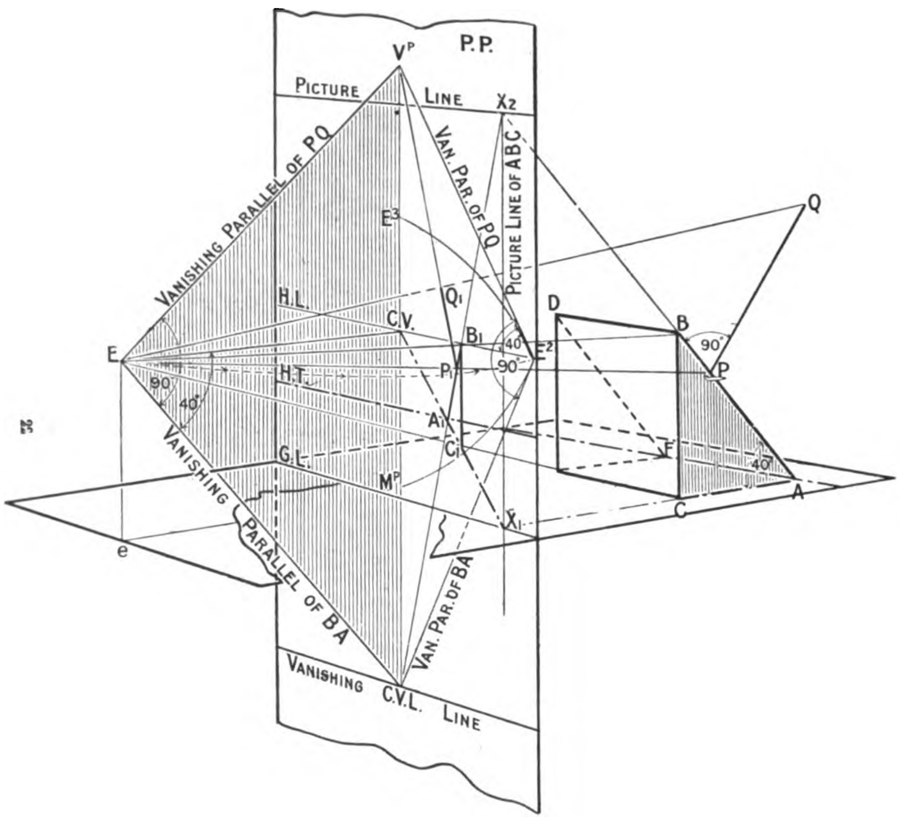


Fig. 21.

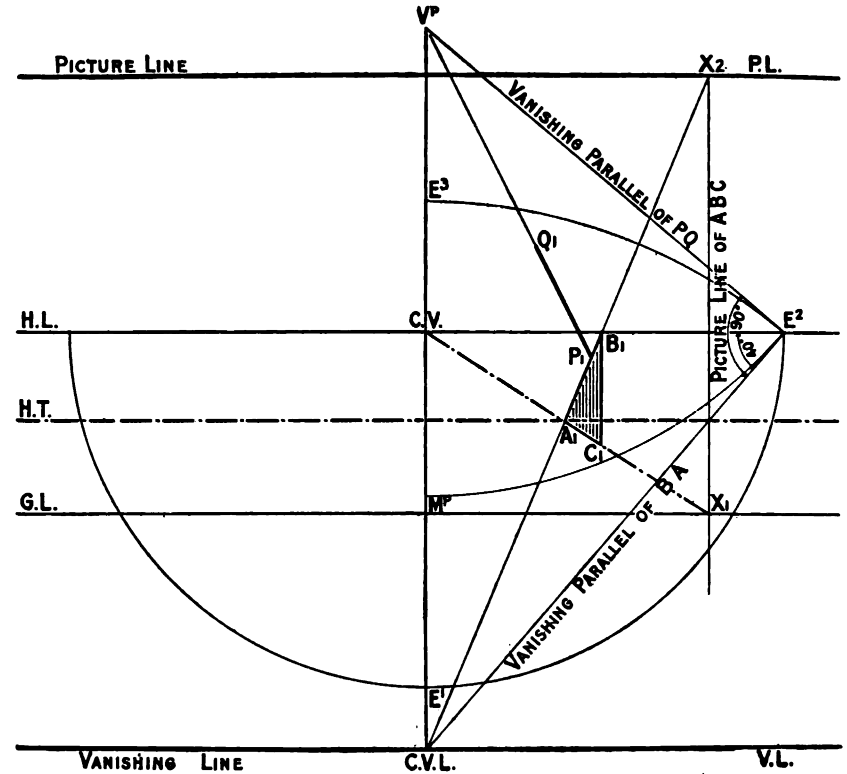


Fig. 21a.

DESCENDING PLANES.

Fig. 21 shows the eye, **P.P.**, and a descending plane **ABDF** in position. **ABDF** is inclined at 40° to the ground, and **AF** is its horizontal trace. **PQ** is a perpendicular to **ABDF** from the point **P** which is on the plane.

Suppose **ABDF** to be cut by a vertical plane **BAC**, containing **P**, so that the horizontal trace **AC** of **BAC** is at right angles to **AF**. **AB** is the intersection of **BAC** with **ABDF**. Observe that **AC** will be at right angles to the **P.P.**, and that **BAF** will be a right angle.

To determine the **V.L.** *of* **ABDF**:—Through **E** draw **EC.V.L.** parallel to **AB**, cutting the **P.P.** at **C.V.L.** Since **ABC** is a vertical plane at right angles to the picture plane, **C.V.L. C.V.** will be vertical. The angle **C.V. E C.V.L.** will be equal to ∠ **BAC**, *i.e.* the inclination of the plane to the ground.

C.V.L. is the **V.P.** of **AB**; the **V.L.** of **ABDF** is obtained by drawing a horizontal line through **C.V.L.**

To determine the **P.L.** *of* **ABDF**:—The vertical plane **BAC** cuts the ground along **CA** and the **P.P.** in a vertical line X_1X_2 through X_1 on the **G.L.**

Since **AB** lies in **BAC** it will cut the **P.P.** at X_2, where **AB** produced cuts X_1X_2.

However, as **AB** also lies on **ABDF**, the **P.L.** of **ABDF** must pass through X_2, and it is horizontal.

To determine the **V.P.** *of the perpendiculars to* **ABDF**:—The vertical plane **BAC** contains **PQ**; **C.V.L. C.V.** V^P is the vanishing line of this plane. **E** V^P is parallel to **PQ** and intersects the **P.P.** at V^P.

V^P is the **V.P.** of lines perpendicular to **ABDF**.

Observe that **C.V.L. E** V^P is a right angle.

Suppose **C.V.L. E** V^P to be rotated into the **P.P.** about **C.V.L.** V^P; **C.V.L.** and V^P would remain fixed but **E** would lie on the **P.P.** at E^2, **C.V.** E^2 being equal to **C.V. E**; the angle **C.V.** E^2 **C.V.L.** would be equal to the inclination of **ABDF** to the **G.P.** **C.V.L.** E^2V^P would be a right angle.

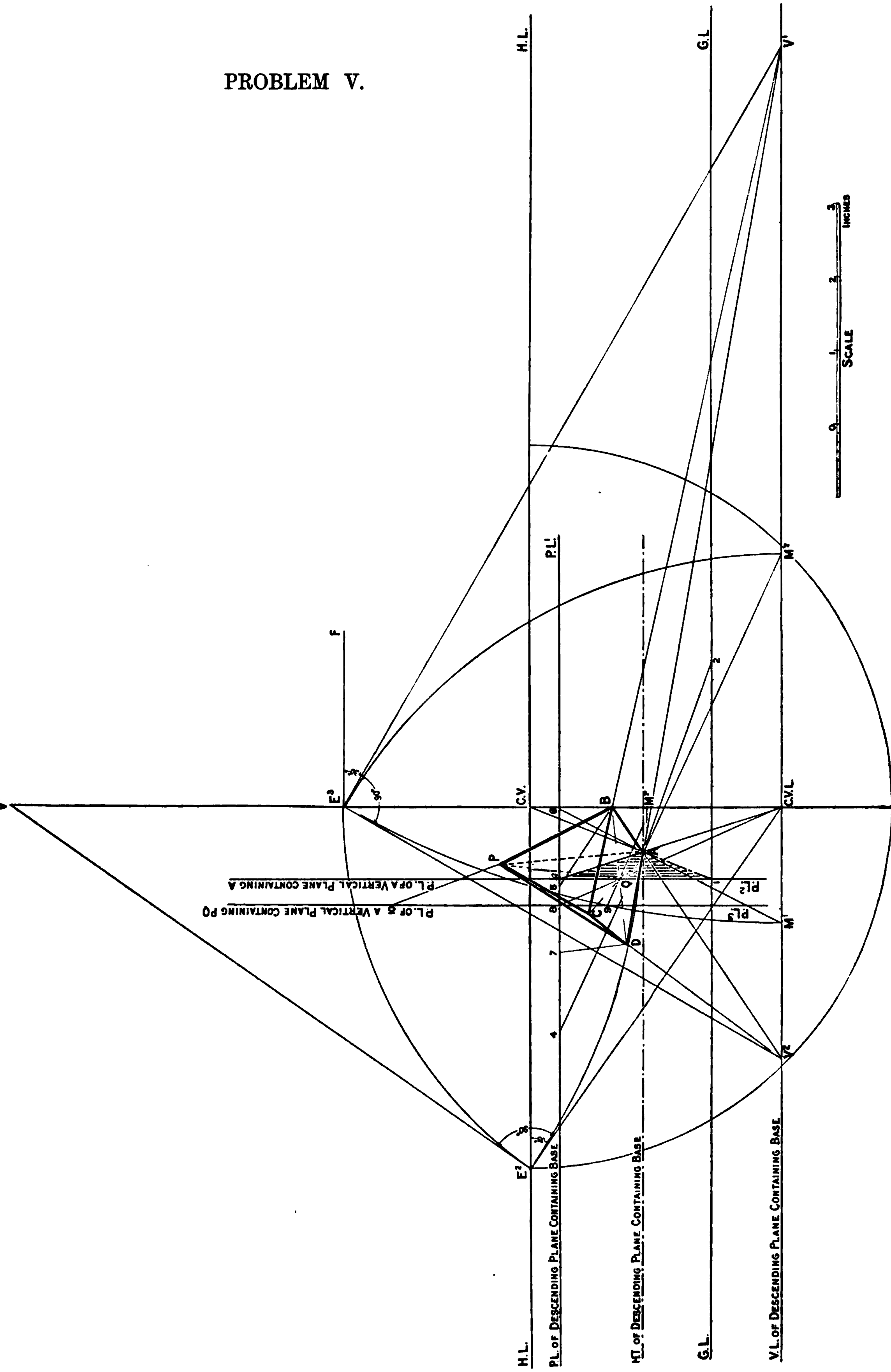
The **M.P.** of V^P, *i.e.* M^P, may be obtained by describing an arc with V^P as centre and V^PE^2 as radius.

If the vanishing plane of **ABDF** was rotated into the **P.P.** about the vanishing line the eye would lie at E^3. **C.V.L.** E^3 being equal to **C.V.L. E**, which is also equal to **C.V.L.** E^2.

Fig. 21*a* is a reproduction of the foregoing construction as it would be executed on a drawing.

The horizontal trace of the plane is a horizontal line passing through **A**.

PROBLEM V.


H.L.
P.L. of Descending Plane Containing Base
H.T. of Descending Plane Containing Base
G.L.
V.L. of Descending Plane Containing Base
P.L. of a Vertical Plane containing A
P.L. of ≅ a Vertical Plane containing PQ
C.V.
C.V.L
E^1
E^2
E^3
V^1
V^2
V^4
M^1
M^2
M^3
$P.L.^1$
$P.L.^2$
$P.L.^3$
F
P
Q
B
C
D
A
90°
Scale
Inches
0
1
2
3

PROBLEM V.

Determine the perspective representation of a square pyramid (side of base 2 ins., axis 3 ins.) when it rests with one corner of the base (**A**) *on the ground 1 in. on the spectator's left and 3 ins. within the picture. The base rests on a descending plane inclined at 35° to the ground, one side making an angle of 30° with the* **H.T.** *of that plane downwards away from the spectator towards his right. The eye is 5 ins. from the* **P.P.** *and 2½ ins. above the ground.*

Find the corner **A**. The horizontal chain line through **A** is the **H.T.** of the descending plane containing the base.

With **C.V.** as centre and **C.V.** E^1 as radius describe an arc of a circle cutting **H.L.** at E^2. Set off the angle **C.V.** E^2 **C.V.L.** equal to the plane's inclination, *i.e.* 35°. Through **C.V.** draw a vertical line to cut E^2 **C.V.L.** at **C.V.L.** A horizontal line through **C.V.L.** is the **V.L.** of the plane of **A B C D.**

Suppose a vertical plane perpendicular to the **P.P.** to contain **A**, the chain line **C.V. A 1** is the intersection of that vertical plane with the ground. **C.V.L. A** is the intersection of the same plane with the descending plane. This vertical plane cuts the **P.P.** in a vertical line **1,3** through **1**. **C.V.L. A** cuts the **P.P.** at **3** in this line.

The **P.L.** of the descending plane is a horizontal line through **3**. Find V^P, the vanishing point of the perpendiculars, and its **M.P.** Consider the eye with the vanishing parallels of **A D** and **A B** to be rotated into the **P.P.** about V^1V^2; the eye would lie at E^3.

The horizontal line E^3F through E^3 represents the vanishing parallel of the **H.T.** of **A B C D** rotated into the **P.P.**

$F\,E^3V^1$ is 30° and $V^1E^3V^2$ is a right angle.

Find M^1 and M^2, the **M.P.**s of V^1 and V^2 respectively.

Produce V^2A; draw M^2A to cut $P.L.^1$ at **4**; make **4,5** 2 ins. long, and join **5** to M^2 cutting $V^2A\,B$ at **B**.

Produce M^1A to cut $P.L.^1$ at **6**. Measure off **6,7** on $P.L.^1$ 2 ins. long, and join **7** M^1 cutting V^1A produced at **D**.

C D and **B C** vanish at V^2 and V^1 respectively.

This completes the square base.

To find the vertex:—Draw the diagonals of the base and let them intersect at **Q**.

Join **Q** to V^P; this line represents the axis.

Suppose a vertical plane to contain the axis, it cuts the descending plane in **C.V.L. Q** produced, and it cuts the **P.P.** in $P.L.^3$, a vertical line through **8** which is the intersection of **C.V.L. Q** with $P.L.^1$, *i.e.* the intersection of **C.V.L. Q** with the **P.P.** Produce $M^P Q$ to cut $P.L.^3$ at **9**. Measure off the length of the axis, *i.e.* 3 ins. from **9** to **10**, and join **10** M^P cutting $Q\,V^P$ at **P**. **P** is the vertex.

Complete the pyramid by joining the vertex to **A, B, C,** and **D**.

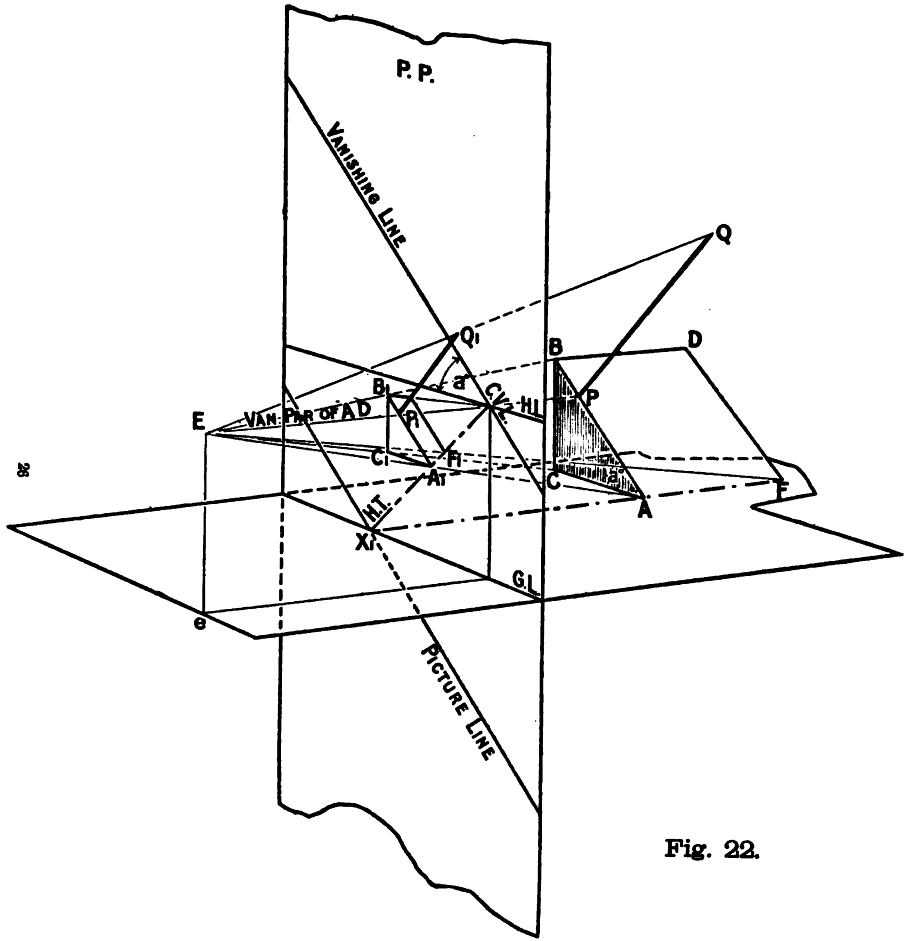


Fig. 22.

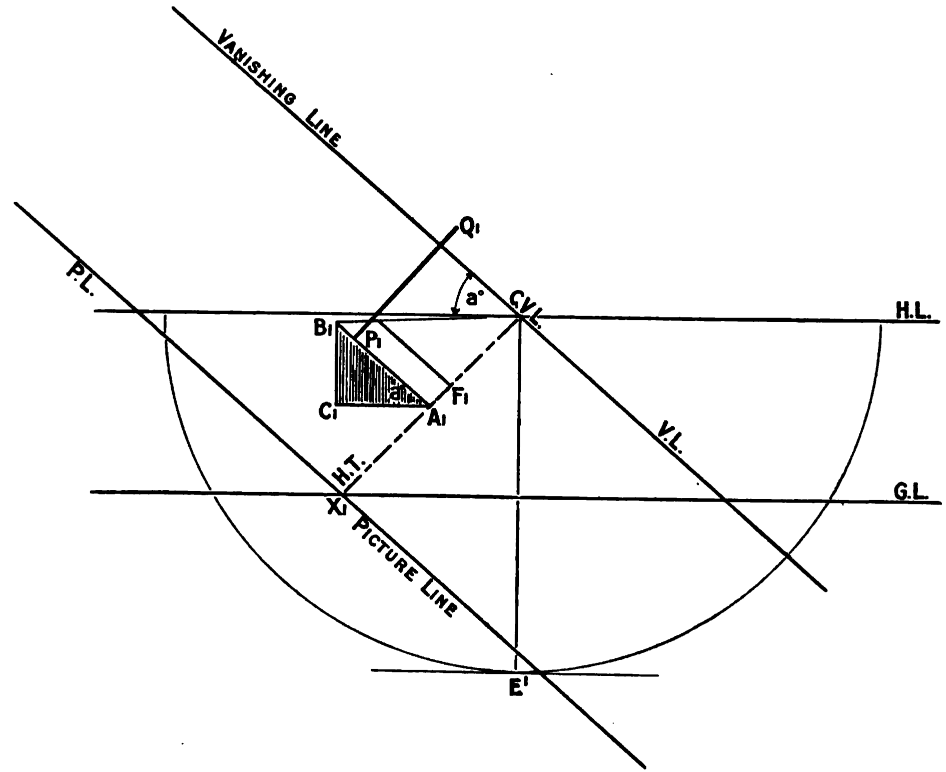


Fig. 22a.

INCLINED PLANES.

Fig. 22 shows the eye, **P.P.**, and an inclined plane **A B D F** in position. **A B D F** is inclined at **a°** to the ground, and **A F** is its horizontal trace. **P Q** is a perpendicular to **A B D F** from the point **P** which is on the plane. **B A C** is a vertical plane cutting **A B D F**, so that the horizontal trace **A C** is at right angles to **A F**; observe that **A C** and **A B** are parallel to the **P.P.**

To determine the **V.L.** *of* **A B D F**:—Through **E** draw **E C.V.L.** parallel to **A F** cutting the **P.P.** at **C.V.L.** **C.V.L.** coincides with the **C.V.**

Since **B A C** is parallel to the **P.P.**, the **V.L.** of **A B F D** will make the same angle with the **H.L.** as **B A** makes with **A C**, *i.e.* **a°**.

Thus, the **V.L.** of an inclined plane is a line drawn through the **C.V.** making the same angle with the **H.L.** as the plane makes with the ground.

To determine the **P.L.** *of* **A B D F**:—**F A** cuts the **P.P.** at X_1, thus the **P.L.** of **A B D F** is a line through X_1 and it is parallel to the **V.L.**

P Q lies in the vertical plane **B A C**, and is parallel to the **P.P.**, hence it will be represented in perspective by a line parallel to itself. Observe that the perspective representation of **A B** is parallel to the **V.L.**, and **P Q** being perpendicular to **A B** will be represented by a line perpendicular to the **V.L.**

Fig. 22*a* is a reproduction of the foregoing construction as it appears on the **P.P.** The horizontal trace of the plane vanishes at the **C.V.**

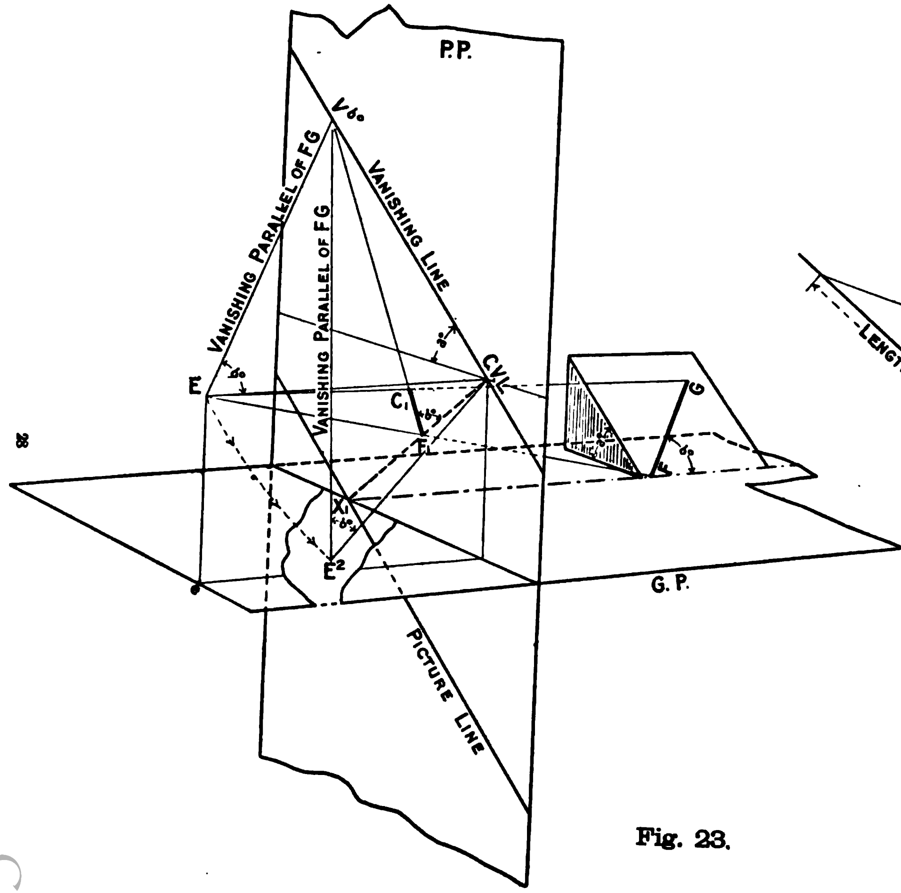


Fig. 23.

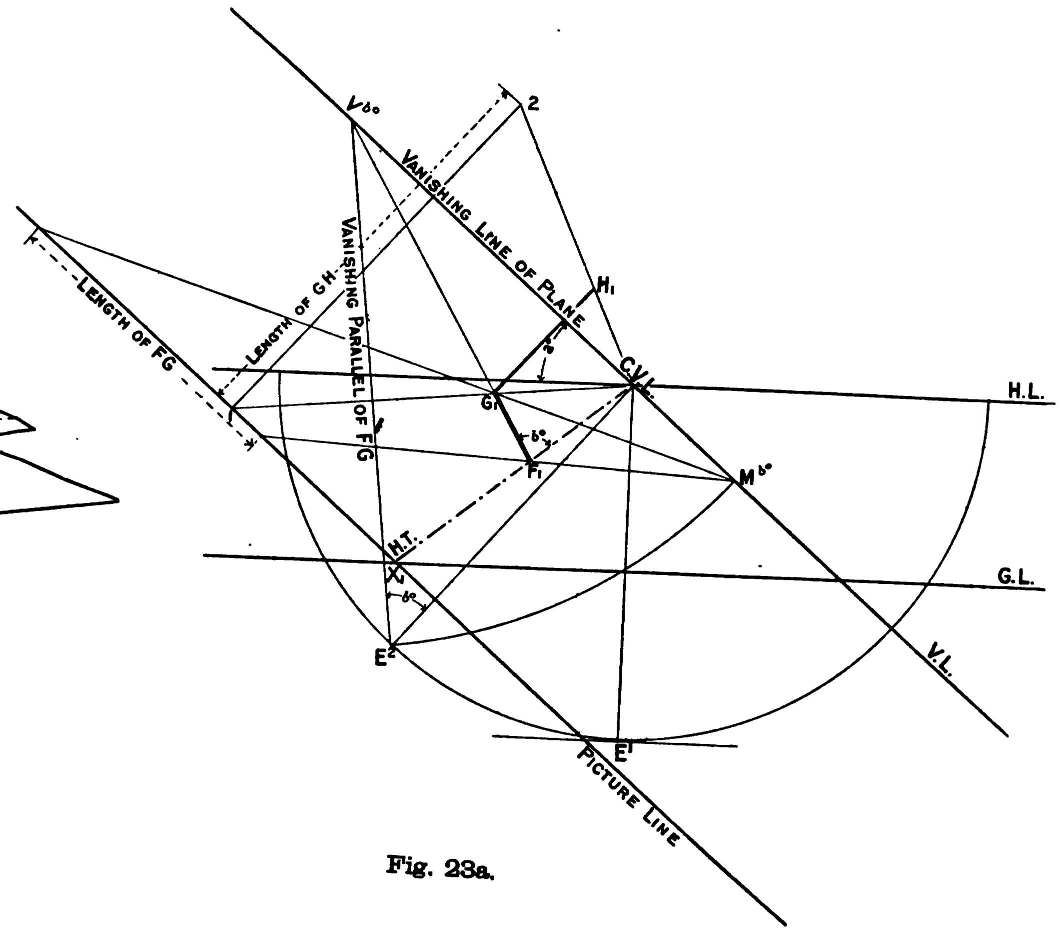
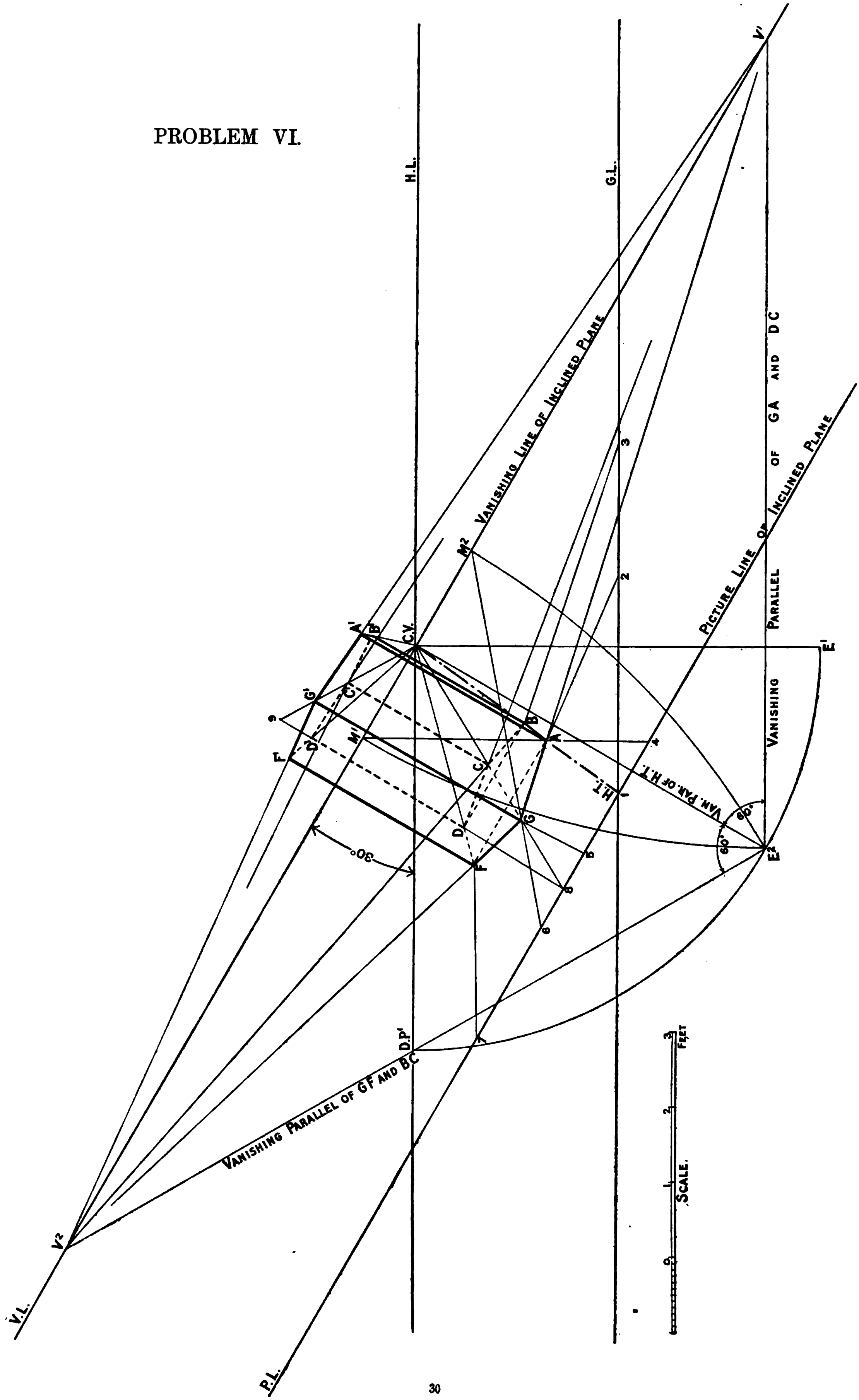


Fig. 23a.

Fig. 23 shows a line **F G** lying on an inclined plane which makes a° with the ground, the line making b° with the **H.T.** of that plane. **E** $V^{b°}$ is the vanishing parallel of **F G** giving $V^{b°}$ as the **V.P.** of **F G**. Suppose the triangle **E** $V^{b°}$ **C.V.L.** to be rotated into the **P.P.** about $V^{b°}$ **C.V.L.**, and let E^2 be the position of the eye after rotation (the arrow heads indicate the path of **E**). $V^{b°}$ **C.V.L.** E^2 is a right angle as $V^{b°}$ **C.V.L. E** is a right angle. **C.V.L.** E^2 is equal to **C.V.L. E**, and **C.V.L.** $E^2 V^{b°}$ is equal to b°, *i.e.* the angle **F G** makes with the **H.T.** of the ascending plane.

In fig. 23*a* the lettering corresponds to fig. 23, and shows the construction required in finding the perspective representation of **F G**. $M^{b°}$ is the **M.P.** of $V^{b°}$, and is obtained by drawing an arc with $V^{b°}$ as centre and $V^{b°}E^2$ as radius cutting the **V.L.** at $M^{b°}$. G_1H_1 is the perspective representation of a perpendicular to the inclined plane and is shown perpendicular to the **V.L.** 1 G_1 **C.V.L.** is the intersection with the inclined plane of a plane containing **G H**, having its **H.T.** perpendicular to the **P.P.** This plane cuts the **P.P.** in **1,2** a line perpendicular to the **P.L.** through **1**. **1,2** is the real length of G_1H_1.

PROBLEM VI.



H.L.
G.L.
V.L.
P.L.
Vanishing Line of Inclined Plane
Picture Line of Inclined Plane
Vanishing Parallel of GA and DC
Vanishing Parallel of GF and BC
Van. Par. of H.T.
H.T.
C.V.
D.P'
30°
60°
Scale.
Feet

PROBLEM VI.

A hexagonal prism (side of hexagon 1 ft. 9 ins., axis 4 ft. 6 ins.) rests with an edge of one of its hexagonal faces on the ground, perpendicular to the **P.P.**; *the nearer end of that edge to the spectator is 2 ft. on the left and 3 ft. from the ground line. Show its perspective representation when its hexagonal faces make 30° with the ground upwards towards the left, the height of the eye above the ground being 2 ft. 9 ins., the distance of the spectator from the* **P.P.** *being 5 ft. 6 ins. Scale 1 in. to 1 ft.*

Find **A** 2 ft. on the left and 3 ft. in; make **A B** perpendicular to the **P.P.** and 1 ft. 9 ins. long.

The **V.L.** of the plane containing the prism's ends makes 30° with the **H.L.** and passes through **C.V.** **A B** cuts the **P.P.** at 1, the **P.L.** is parallel to the **V.L.** and passes through 1.

E^2 is the position of the eye, and **E**^{2} **C.V.** is the vanishing parallel of **A B** when rotated into the **P.P.** about the **V.L.** **E**^{2} **V**^{1} and **E**^{2} **V**^{2} are the vanishing parallels of **G A** and **B C** respectively, **C.V. E**^{2} **V**^{1} and **C.V. E**^{2} **V**^{2} must each be 60°. **M**^{1} and **M**^{2} are the **M.P.**s of **V**^{1} and **V**^{2} respectively.

Produce **V**^{1} **A**; join **M**^{1} **A** and produce to cut the **P.P.** at **4** on the **P.L.**; measure off **4,5** 1 ft. 9 ins. long; join **5 M**^{1} cutting **V**^{1} **A** at **G**.

Similarly obtain **G F** but use **M**^{2}.

A F should be parallel to the **P.L.** The hexagon may easily be completed as **G C** and **F D** vanish at **C.V.**, and **B D** is parallel to **A F**.

To measure the height:—**C.V. C G 8** is the intersection with the plane of the hexagon of a plane containing the edge **G G**^{1}. This plane cuts the **P.P.** in **8,9**, through **8** perpendicular to the **P.L.** **8,9** is 4 ft. 6 ins. long (the length of the prism's axis), this determines **G**^{1} and **C**^{1} as the long edges are all at right angles to the **V.L.** The upper face can now easily be completed.

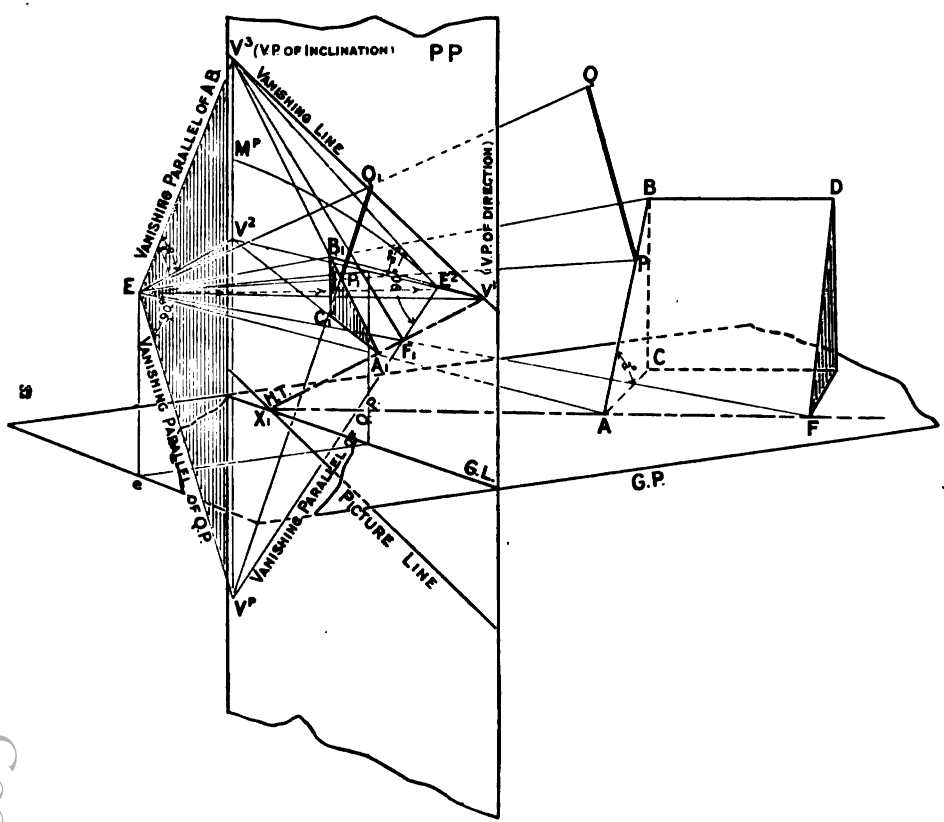


Fig. 24.

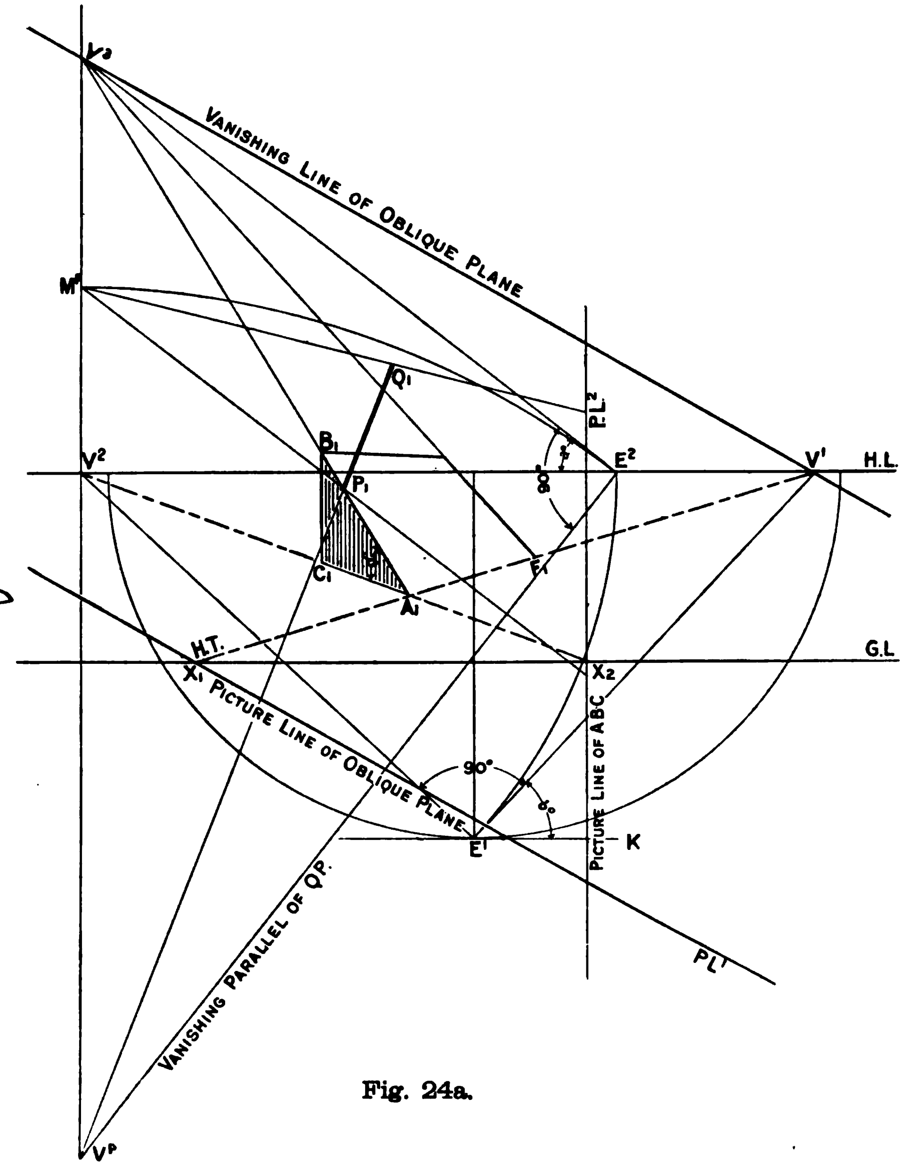


Fig. 24a.

OBLIQUE PLANES.

Fig. 24 shows the eye, **P.P.**, and an oblique plane **A B D F** in position. **A B D F** is inclined at **a**° to the ground, and its horizontal trace **A F** recedes from the **P.P.** towards the right. **P Q** is a perpendicular to **A B D F** from the point **P**, which is on the plane.

Suppose **A B D F** to be cut by a vertical plane **B A C** containing **P**, so that the horizontal trace **A C** of **B A C** is at right angles to **A F**. **A B** is the intersection of **B A C** with **A B D F**. Observe that ∠**B A C** = **a**° and ∠**F A C** = 90°.

To determine the **V.L.** *of* **A B D F**:—The **V.L.** of a plane is always a straight line, hence the position of the **V.L.** is determined when *two points* in it are found. In practice the two points which are generally found on the **V.L.** of an oblique plane correspond to **A F** and **A B**, *i.e.* the plane's **H.T.** and a line on the plane perpendicular to that **H.T.**

EV^1 is parallel to **A F**, and gives V^1 on the **H.L.** as the **V.P.** of **A F**. $V^3V^2V^P$ is the vanishing line of the vertical plane **B A C**, V^3 being the **V.P.** of **A B**. V^1V^3 is the **V.L.** of **A B D F**. EV^P is parallel to **Q P**, and cuts the **P.P.** at V^P, the **V.P.** of the perpendiculars to the plane.

Observe that V^1EV^2 is a right angle. V^3EV^2 is **a**° and V^3EV^P is a right angle.

Rotate V^3EV^P into the **P.P.** about V^3V^P, the path of **E** is indicated by arrow-heads until it meets the **P.P.** at E^2. $V^2E^2V^3$ is now **a**°, $V^3E^2V^P$ a right angle, and $V^2E^2 = V^2E$. The **P.L.** is a line through X_1 (the intersection of **F A** with the **P.P.**), parallel to the **V.L.**

Fig. 24*a* is a reproduction of the foregoing construction as it is employed in working a problem.

KE^1V^1 is **b**°, *i.e.* the angle that the plane's **H.T.** makes with the **P.P.** V^1 is the **V.P.** of direction.

$V^1E^1V^2$ is a right angle. $V^3V^2V^P$ is the **V.L.** of the vertical plane **B A C**. V^3 is obtained by making $V^2E^2V^3$ = the plane's inclination to the ground (in this case **a**°).

M^P is obtained by measuring off $V^PM^P = V^PE^2$.

The vertical plane **B A C** contains the perpendicular **P Q**, the horizontal trace of **B A C**, *i.e.* **C A**, cuts the **P.P.** at X_2, hence **B A C** would cut the **P.P.** in a vertical line **P.L.**² through X_2. **P.L.**² and M^P would be used for measuring lengths along **P Q**.

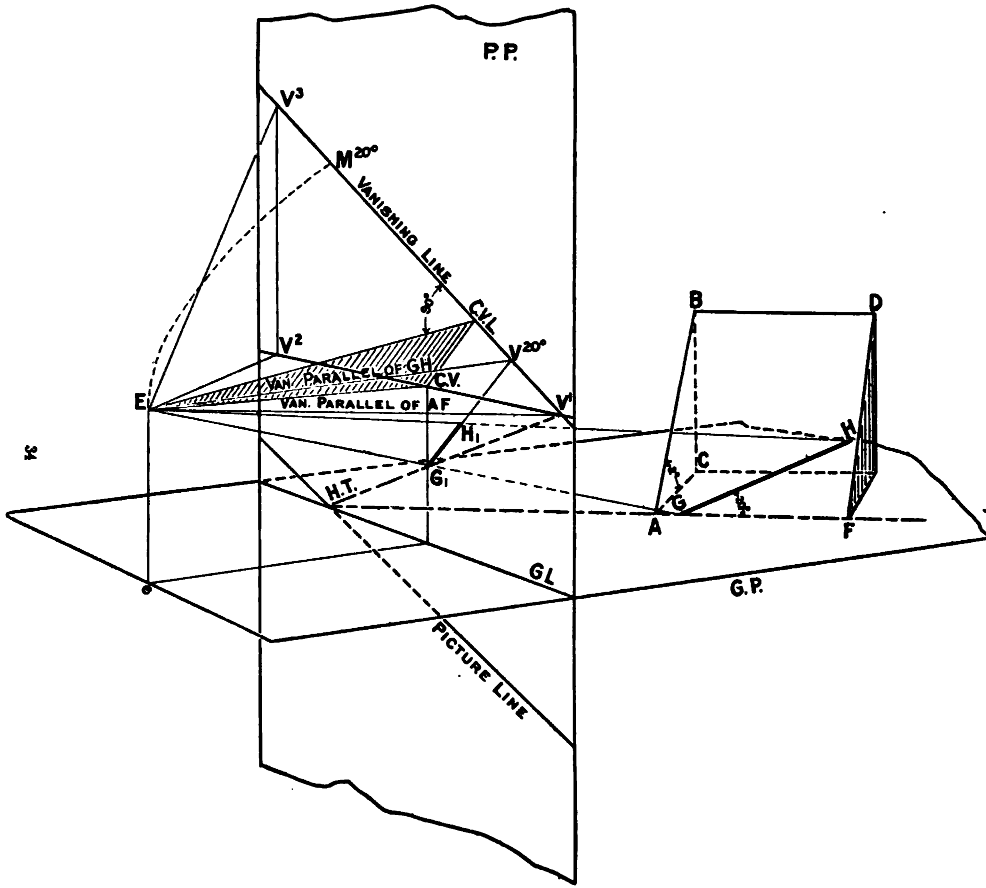


Fig. 25.

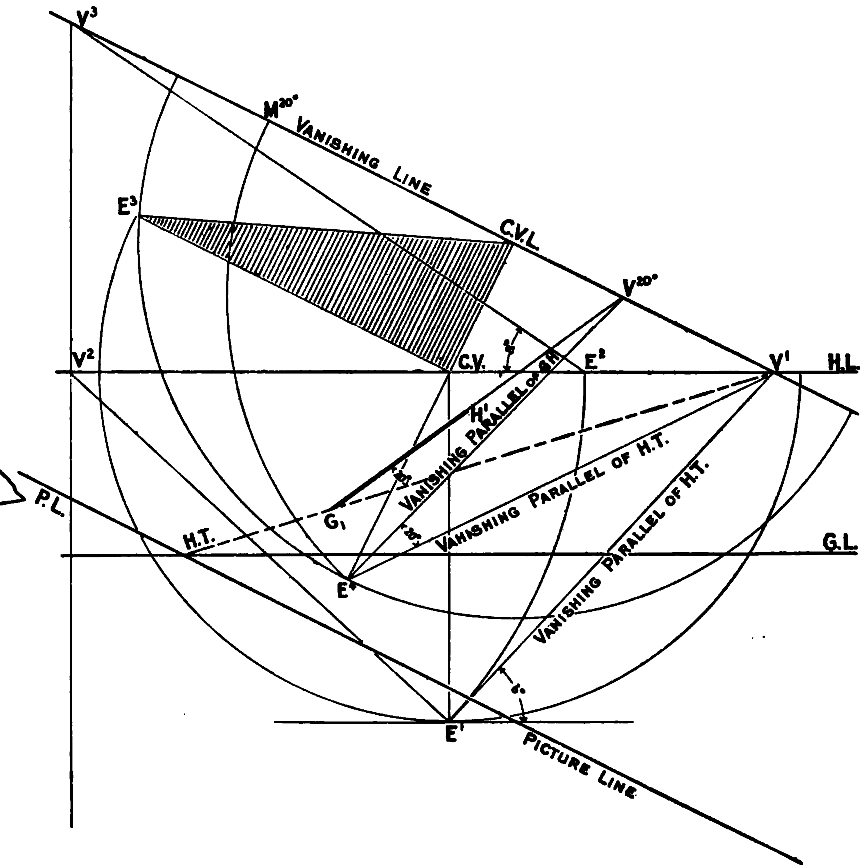


Fig. 25a.

OBLIQUE PLANES—*Continued.*

Fig. 25 shows the eye, **P.P.**, and an oblique plane **A B D F** in position. **G H** is a line lying on **A B D F**, beginning at **G**, and makes 20° *upwards* away from the spectator with **A F**. (There is also a line, which is not shown, having one end at **G**, making 20° with **A F** inclined *downwards* away from the spectator.)

$V^3 V^1$ is the vanishing line of **A B D F**. **E C.V.L.** is a perpendicular to the **V.L.** and determines the **C.V.L.** $E\,V^{20^\circ}$ is the vanishing parallel of **G H**, and as $E\,V^1$ is the vanishing parallel of **A F**, therefore $V^{20^\circ}\,E\,V^1$ is 20°.

The position of the eye:—Consider the shaded triangle **C.V.L. C.V. E** in fig. 25 to rotate into the **P.P.** about **C.V.L. C.V.** **C.V.L. C.V.** remains fixed, and as **C.V.L. C.V. E** is a right angle, therefore after rotation the eye will appear at E^3 (fig. 25*a*). The angle **C.V.L.** **C.V.** E^3 being 90° and thus parallel to the **V.L.** **C.V.** E^3 is equal to **C.V.** E^1, thus **C.V.L.** E^3 is equal to the distance from **C.V.L.** to **E**.

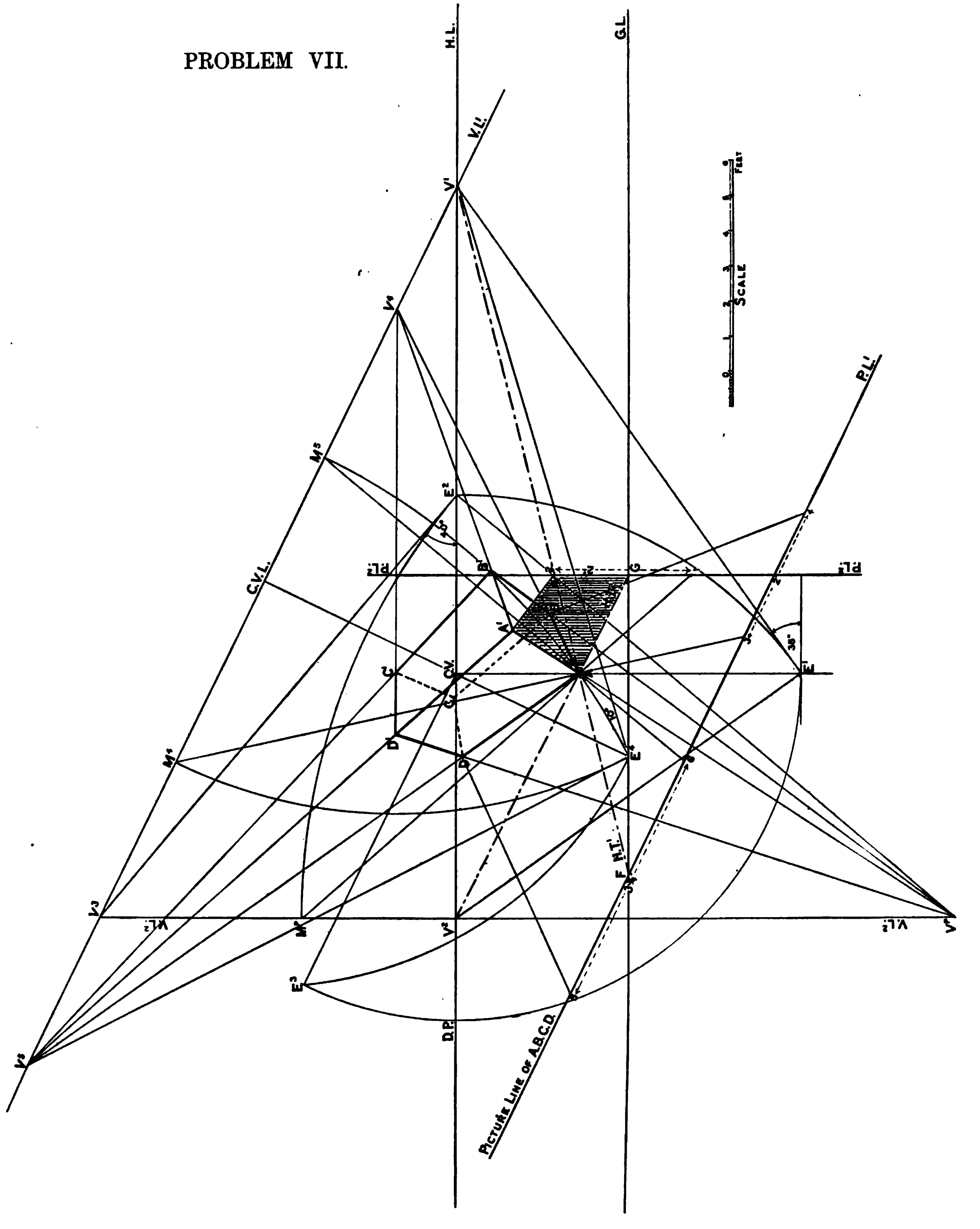
Now rotate the triangle $V^3 E\,V^1$ with **C.V.L. E** and $V^{20^\circ}E$ upon it into the **P.P.**

As V^3**C.V.L. E** is a right angle, after rotation **C.V.L.** E^4 will be at right angles to the **V.L.**, and it has been shown to be equal to **C.V.L.** E^3; thus E^4 is the eye's new position. E^4V^1 is $E\,V^1$ after rotation as V^1 remains fixed; the angle $V^1E^4V^{20^\circ}$ is 20° as $V^1E\,V^{20^\circ}$ is 20°. $E^4V^{20^\circ}$ is equal to $E\,V^{20^\circ}$, hence M^{20° can be obtained by making $V^{20^\circ}M^{20^\circ}$ equal to $E^4V^{20^\circ}$.

Lengths are measured on the **P.L.** of **A B D F**, as **G H** lies on **A B D F**.

The perspective representation of the line from **G**, making 20° with **A F** inclined *downwards*, would be obtained by finding on the vanishing line of **A B D F** the **V.P.** and **M.P.** of the required line, the **V.P.** being found by drawing its vanishing parallel at 20° to E^4V^1 on the right.

PROBLEM VII.



PROBLEM VII.

A square prism (side of square 4 ft., axis 7 ft. 6 ins.) has one corner (A) *opposite the spectator 4 ft. from the* P.P. *on a horizontal plane 5 ft. below the eye. The prism rests with one of its largest faces on an oblique plane inclined at 40° to the ground, the* H.T. *of that plane receding from the* P.P. *at 35° towards the right. Show its perspective representation when one of the 4-ft. edges makes 10° upwards away from the spectator with the* H.T. *of the oblique plane. Distance from the* P.P. *to the spectator 10 ft. Scale ½ in. to 1 ft.*

Find A in the given position. E^1V^1 the vanishing parallel of the H.T. of the oblique plane makes 35° towards the right. E^1V^2 is at right angles to E^1V^1. $V^3V^2V^P$ is a vertical line through V^2, and represents the V.L. of all vertical planes cutting the oblique plane at right angles to its H.T.

Rotate the eye into the P.P. at E^2. $V^2E^2V^3$ is 40° the plane's inclination. V^1V^3 is the V.L. of oblique plane on which the prism rests.

C.V. E^3 is equal to C.V. E^1, and is parallel to V.L.^{1}.

C.V.L. E^4 is at right angles to V.L.^{1} through C.V., and is equal to C.V.L. E^3.

Join E^4V^1. E^4 is the position of the eye, and E^4V^1 is the vanishing parallel of H.T.^{1} when rotated into the P.P. about V.L.^{1}.

N.B. In measuring along a line, great care must be taken that the P.L. used is that of a plane *containing the line.*

$V^1E^4V^4$ is 10°, and is the vanishing parallel of A B. $V^4E^4V^5$ is 90° as B A D is 90°.

M^4 and M^5 are obtained by taking V^4 and V^5 as centres and V^4E^4 and V^5E^4 as radii and cutting V.L.^{1}.

As A is on the ground, P.L.^{1} is obtained by producing V^1A to cut the G.L. at F and drawing P.L.^{1} parallel to V.L.^{1}.

V^P is obtained by setting off 90° at $V^3E^2V^P$.

With V^P as centre and V^PE^2 as radius draw an arc cutting V^3V^P at M^P.

Obtain A B and A D in perspective by joining A to V^4 and V^5 and measuring off the required lengths, using M^4 and M^5 as M.P.s and P.L.^{1} as the picture line. Complete A B C D.

Suppose a vertical plane to contain A A^1, it cuts the ground in V^2A (H.T.^{2}), and H.T.^{2} cuts the P.P. at G on the G.L. Hence the imaginary plane cuts the P.P. in a vertical line through G (P.L.^{2}).

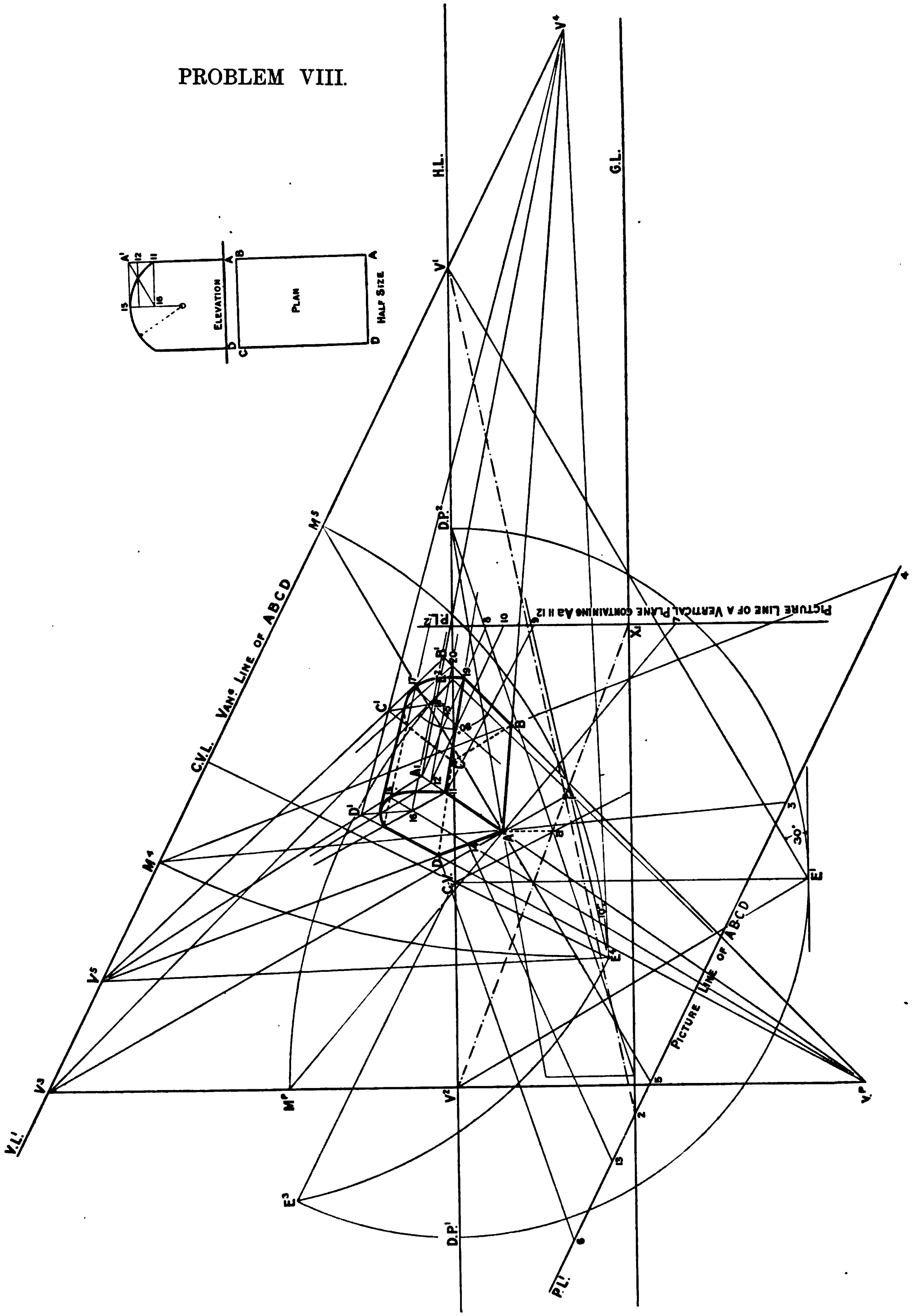
Measure off A A^1 4 ft. long, using M^P and P.L.^{2}.

Observe that A A^1 is the *only* edge of the prism that the assumed vertical plane contains.

A^1B^1 vanishes at V^4 and B B^1 at V^P. A^1D^1 vanishes at V^5 and DD^1 at V^P. This determines B^1 and D^1.

Complete the prism as shown.

PROBLEM VIII.



PROBLEM VIII.

(From a Recent Examination Paper.)

The plan and elevation are given of a box form with carved lid. Draw the box in perspective, the under surface **A B C D** *resting on a plane which recedes upwards at an angle of 45° with the ground, while its horizontal trace vanishes at 30° towards the right. The point* **A** *is 1 ft. 3 ins. above the ground, 1 ft. 3 ins. to the right of the spectator, and 4 ft. within the picture. The edge* **A B** *recedes downwards, and makes an angle of 10° with the horizontal trace of the oblique plane. The eye is to be 5 ft. from the* **P.P.**, *and 2 ft. 6 ins. above the ground plane. Scale 1 inch to 1 ft.*

Find **A** 1 ft. 3 ins. on the right, 4 ft. in and 1 ft. 3 ins. above the ground. **a** is the point on the ground beneath **A**.

V^1 is the **V.P.** of direction and V^8 is the **V.P.** of inclination, $\angle V^8E^2V^2$ being 45°. V^8V^1 is the **V.L.** of **A B C D** (**V.L.**^{1}).

Rotate the eye into the **P.P.** at E^4. E^4V^1 is the vanishing parallel of the oblique plane's **H.T.** after rotation.

Set off $V^1E^4V^4$ downwards and equal to 10° (as **A B** recedes downwards at 10° to the **H.T.** of **A B C D**). E^4V^5 is at right angles to E^4V^4. E^4V^4 and E^4V^5 are the vanishing parallels of **A B** and **A D** respectively

Find M^4 and M^5.

To determine the **H.T.** *of* **A B C D**:—Suppose a vertical plane to contain **A** and cut the oblique plane at right angles to the **H.T.** of **A B C D**.

This plane cuts the ground in V^2**a 1** and the oblique plane in V^3**A 1**, thus **1** is a point on the **H.T.** The **H.T.** passes through **1** and vanishes at V^1; it cuts the **P.P.** at **2** on the **G.L.**, hence **2** is a point in the **P.L.** of **A B C D**.

Through **2** draw **P.L.**^{1} parallel to **V.L.**^{1}.

Draw the base **A B C D**, using **P.L.**^{1}, M^4, M^5, V^4, and V^5.

Make $V^8E^2V^P = 90°$. E^2V^P is the vanishing parallel of the perpendiculars (rotated into the **P.P.**), and V^P on V^3V^2 produced is their **V.P.** M^P is the **M.P.** of the perpendiculars.

A vertical plane containing **A A**^{1} cuts the ground at V^2**a** X_1 and the **P.P.** in a vertical line **P.L.**^{2} through X_1.

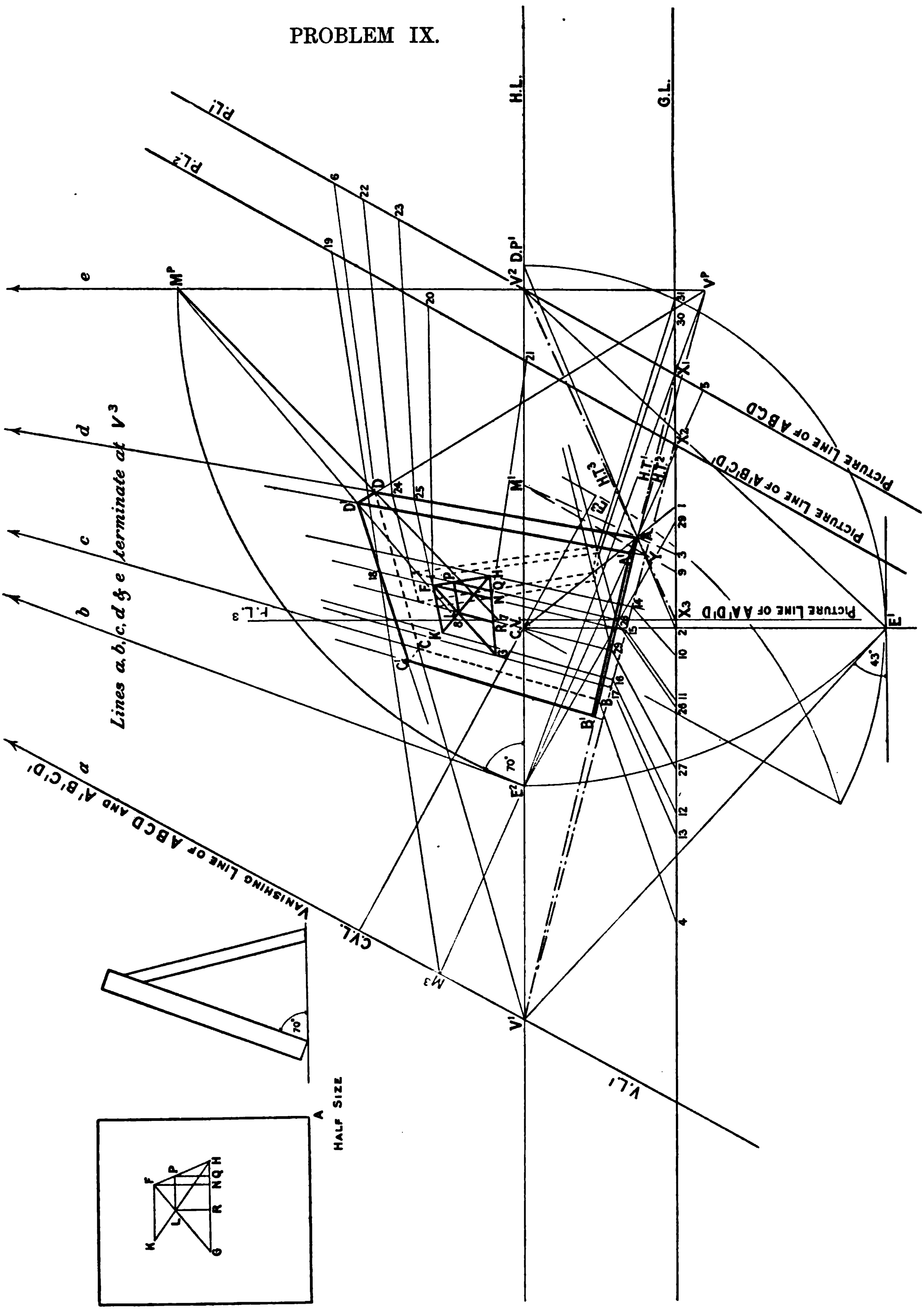
Use M^P and **P L**^{2} to measure **A, 11, 12,** and **A**^{1} corresponding to **A, 11, 12,** and **A**^{1} in the elevation.

11, 12, and **A**^{1} vanish at V^5.

14 is the middle point of **A D**. V^P**14** passes through **15** (where the arc touches the circumscribing rectangle). **A**^{1}**16** and **D**^{1}**16** cut **12** V^5 at two points on the arc.

The more remote face is easily obtained from the first one. Notice that the curved top meets **A B 19,11** in a line **11,19**.

PROBLEM IX.



H.L.
G.L.
P.L.¹
P.L.²
Lines a, b, c, d & e terminate at V³
Vanishing Line of ABCD and A'B'C'D'
C.V.L.
V.L.¹
Picture Line of ABCD
Picture Line of A'B'C'D'
Picture Line of AA'D'D
F.L.³
Half Size

PROBLEM IX.

(From a Recent Examination Paper.)

The scale to be used in working this problem is $\frac{3}{4}$ in. to 1 ft. The eye is to be 6 ft. from the picture plane and 2 ft. 6 ins. above the ground plane.

The front and side elevations are given of a black-board, with a strut behind it, and upon the face of the board a geometrical drawing. The strut, made a foot wide, may be shown in dotted lines as if the board were transparent. Represent the board in perspective with the figure on it; the point **A** *being on the ground 2 ft. to the right of the centre and 2 ft. within the picture, and the edge of the board vanishing towards your left; the face of the board to form an oblique plane making an angle of 70° to the horizontal plane, and the edge on the ground receding at 43° towards the left.*

Find **A B**, the edge on the ground, in the given position. **A B** lies on the **H.T.** of an oblique plane containing the back surface of the board.

V.L.^{1} is the vanishing line of **A B C D** and **$A^1B^1C^1D^1$**. The plane containing **A B C D** cuts the ground along **V^1A** and the **P.P.** at **P.L.**^{1} through **X_1** parallel to **V.L.**^{1}.

The plane of **$A^1A\,D\,D^1$** cuts the ground along **V^2A** and the **P.P.** in a vertical line **P.L.**^{3} through **X^3**.

D^1A^1 cuts the ground at **Y** on **V^2AX_3**, **V^1YX_2** is the **H.T.** of **$A^1B^1C^1D^1$**, and **P.L.**^{2} through **X_2** parallel to **V.L.**^{1} is the **P.L.** of **$A^1B^1C^1D^1$**.

The thickness of the board has been measured at **$D\,D^1$**, as **$A\,A^1$** in this problem is unsuitable.

The figure on the board:—**14**, **15**, **16**, **17** are the intersections with the ground of lines through **H**, **F**, **K**, and **G** respectively, perpendicular to the **H.T.** of **$A^1B^1C^1D^1$**, hence they vanish at **V^3** in perspective.

F N is measured off, using **P.L.**^{2} and **M^3**.

G N H vanishes at **V^1**, and is terminated by **17 V^3** and **14 V^3**.

F K vanishes at **V^1**, and is terminated by **16 V^3**.

Join **F G**, **F H**, and **K H**.

Let **K H** cut **F G** at **L**, then **L P** vanishes at **V^1**.

P Q and **L R** vanish at **V^3**.

This completes the drawing on the board.

P.L.^{1} is used when putting in the strut.

SHADOWS.

Shadows may be caused by sunlight or artificial light. In the former the source of illumination is considered infinitely distant, whereas in the latter the "source" is considered accessible.

The following rules are applicable to sunlight and artificial light, and the working out of the various problems will be greatly facilitated by their being committed to memory.

1. *To determine the shadow of a point on a plane*:—Draw from the "source" the ray which passes through the point; the intersection of this ray with the plane is the required shadow.

2. *To determine the shadow of a line on a given plane*:—Obtain the plane containing the "source" and the line; the line of intersection of this plane with the given plane will contain the line's shadow.

3. If a point is situated on a line its shadow is on the shadow of the line.

4. The shadow of a line on a plane always runs towards the intersection of the line with the plane.

5. The shadow of a line, on a plane parallel to itself, is a line parallel to the line which casts the shadow.

6. The "vanishing point of the shadow of a line" (V_S) is obtained by drawing, through the "source", a line parallel to the line whose shadow is desired, and determining this line's intersection with the plane upon which the shadow is cast.

6*a*. Rule 6, when applied to the shadows cast by the sun, may be stated thus:—

To obtain the **V.P.** *of a line's shadow*:—Join the **V.P.** of the sun to the **V.P.** of the line, the intersection of this line with the **V.L.** of the plane upon which the shadow is cast determines the **V.P.** of the shadow of the line on that plane. (This rule is very important.)

SHADOWS CAST BY THE SUN.

The sun is situated at such a great distance from the earth that the rays proceeding from it to any ordinary object are considered to be parallel. It has already been shown that parallel lines when drawn in perspective vanish at the same point (unless lines parallel to the **P.P.**); thus we proceed to find the **V.P.** of the sun's rays (**S**) by finding the intersection with the **P.P.** of the vanishing parallel of the rays.

The sun may occupy three positions with reference to the **P.P.**, each of which involves a different construction in solving a problem; it may be:

(*a*) behind the **P.P.**, (*b*) in the **P.P.**, or (*c*) in front of the **P.P.**

It should be observed that when the sun is said to be *behind the spectator* it has the same meaning as *in front of the Picture Plane.*

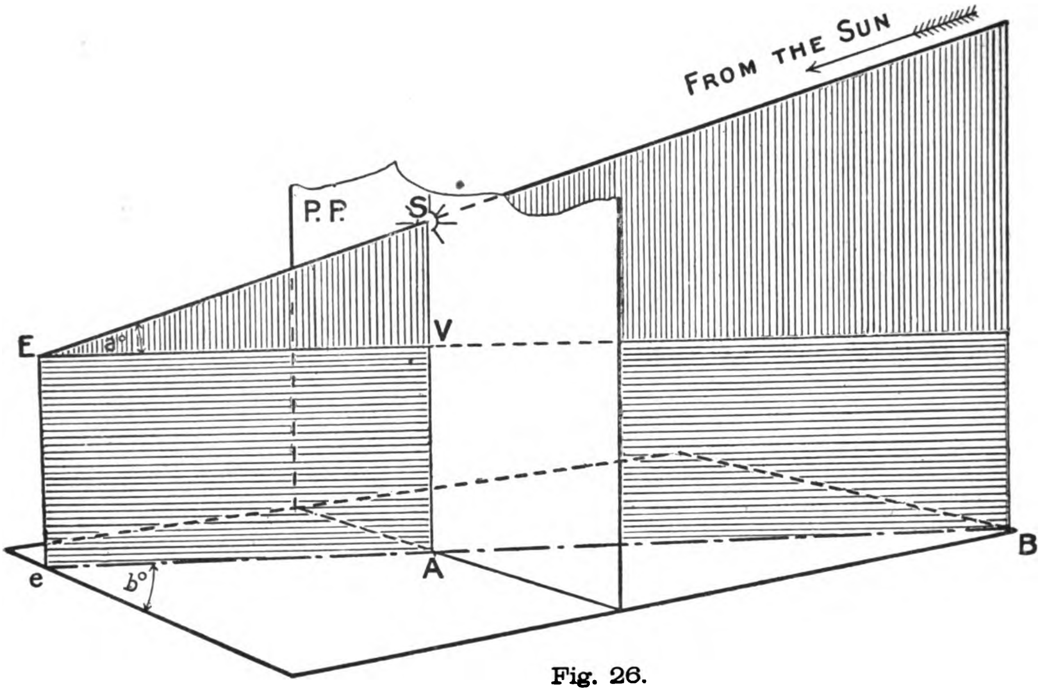


Fig. 26.

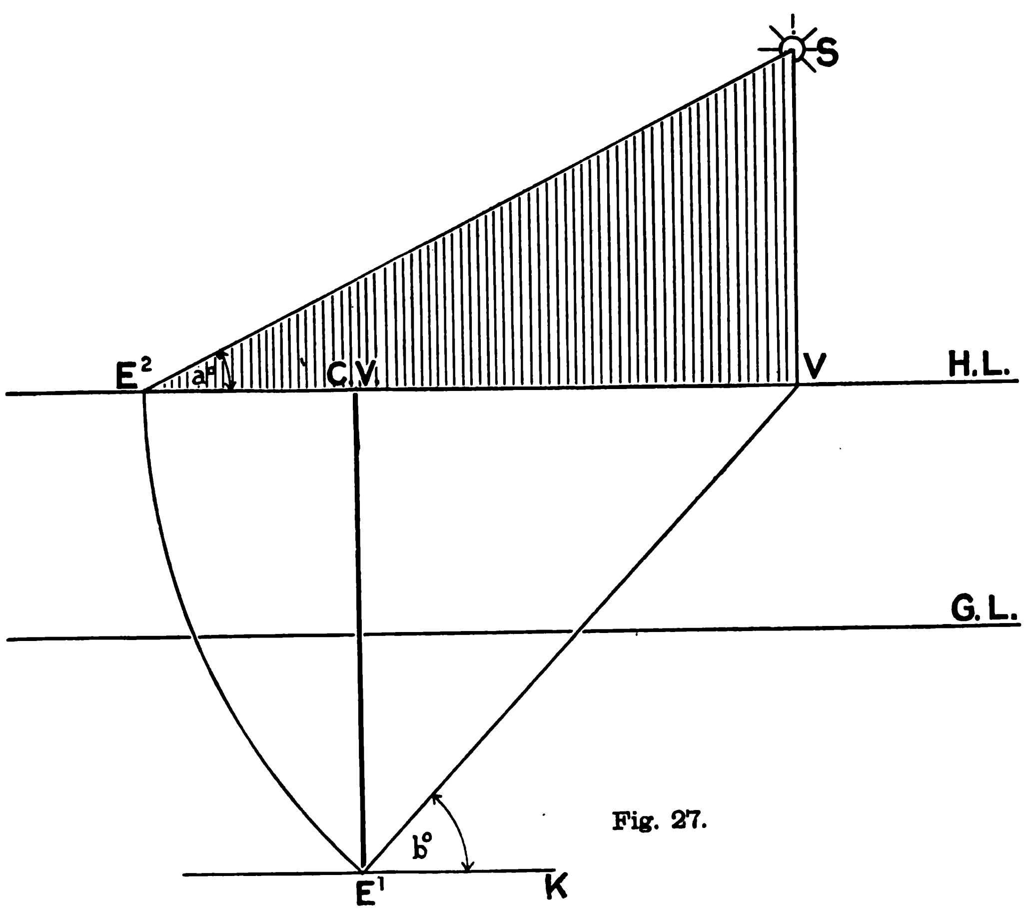


Fig. 27.

To find the **V.P.** *of the sun's rays when the sun is behind the* **P.P.**:—In fig. 26 **E** represents the eye and **P.P.** the picture plane. **E S** is the ray of light from the sun which passes through the eye of the spectator, this line is therefore the vanishing parallel of all the rays, hence **S** is the **V.P.** of the sun's rays.

(Notice that **S** is the perspective representation of the sun, and that all the rays will appear to vanish at **S**.)

E e A B S is a vertical plane containing the sun, **S A** is the **V.L.** of this plane, and $\angle$**V E S** = the altitude of the sun in the same plane.

Suppose **E S V** (fig. 26) to be rotated into the **P.P.** about **S V**, then fig. 27 will be obtained.

The $\angle$**V E**[1]**K** = the angle which the vertical planes containing the rays make with the **P.P.** and $\angle$**S E**[2]**V** = the sun's altitude.

Observe that *when the sun is behind* the **P.P.** and on the spectator's *right*, the **V.P.** of its rays is *above* **H.L.** and on the *right*.

When the sun is in the **P.P.** the **V.P.** of its rays are inaccessible, and are therefore denoted by parallel lines, the direction of a ray generally being given, as shown in fig. 28.

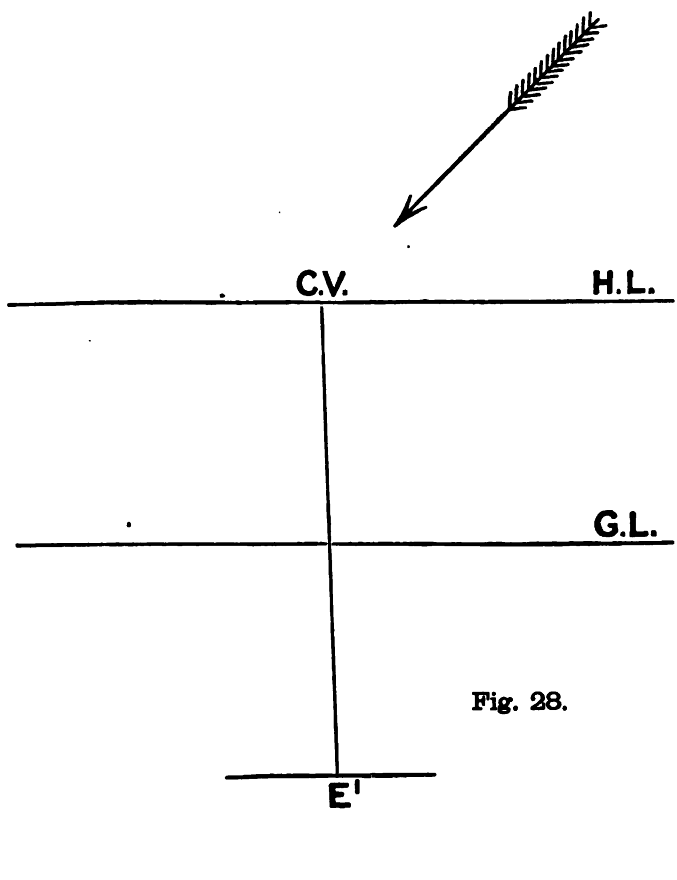


Fig. 28.

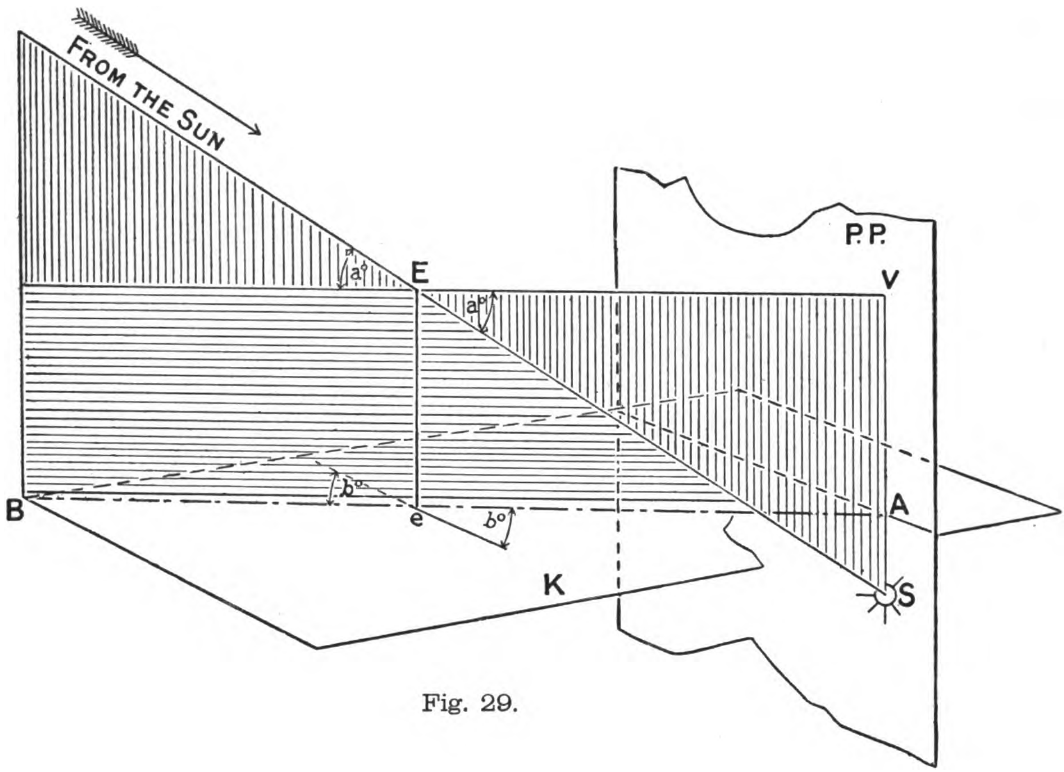


Fig. 29.

To determine the **V.P.** *of the sun's rays when the sun is in front of the* **P.P.**

In figs. 29, 30 the lettering, &c., corresponds to that employed when the sun was behind the **P.P.** Notice that **S** comes below the **H.L.**, the $\angle$**VE²S** (fig. 30) being = the sun's altitude, and that when the sun is *behind* and on the *left* of the spectator, the **V.P.** of its rays is *below* **H.L.** and on the *right.*

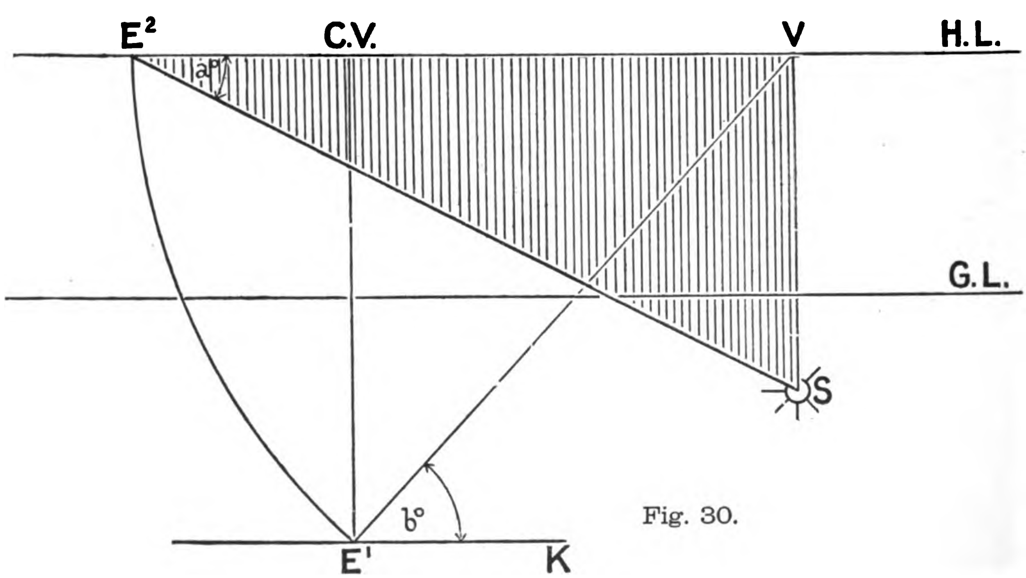
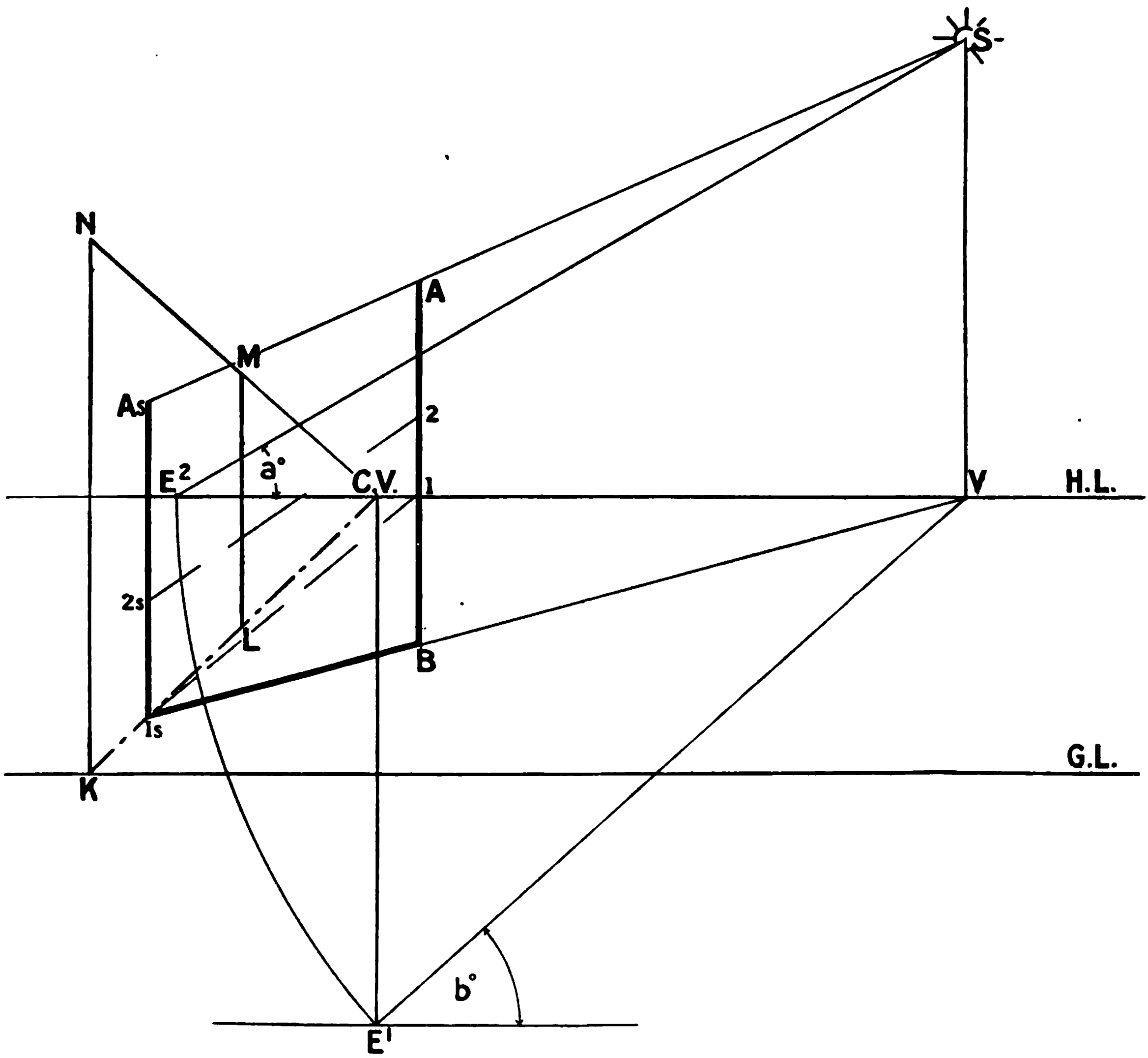


Fig. 30.

SHADOWS CAST ON PLANES BY LINES.

(*a*) On a *vertical plane*, and on a *horizontal plane* by a *vertical line.*
(*b*) On a *vertical plane*, and on a *horizontal plane* by a *horizontal line.*
(*c*) On a *vertical plane*, and on a *horizontal plane* by an *oblique line.*
(*d*) On an *oblique plane*, a *horizontal plane*, a *vertical plane*, and the *ground plane* by an *oblique line.*
(*e*) On an *ascending plane*, a *horizontal plane*, a *plane parallel to* **P.P.**, and on the *ground plane* by an *oblique line.*

In Problems X., XI., and XII., **K L M N** is a vertical plane whose vanishing line is **C.V.E**^{1}. **A B** is a vertical line with the end **B** on the ground. Find the shadow of **A B** on the ground and on **K L M N.**



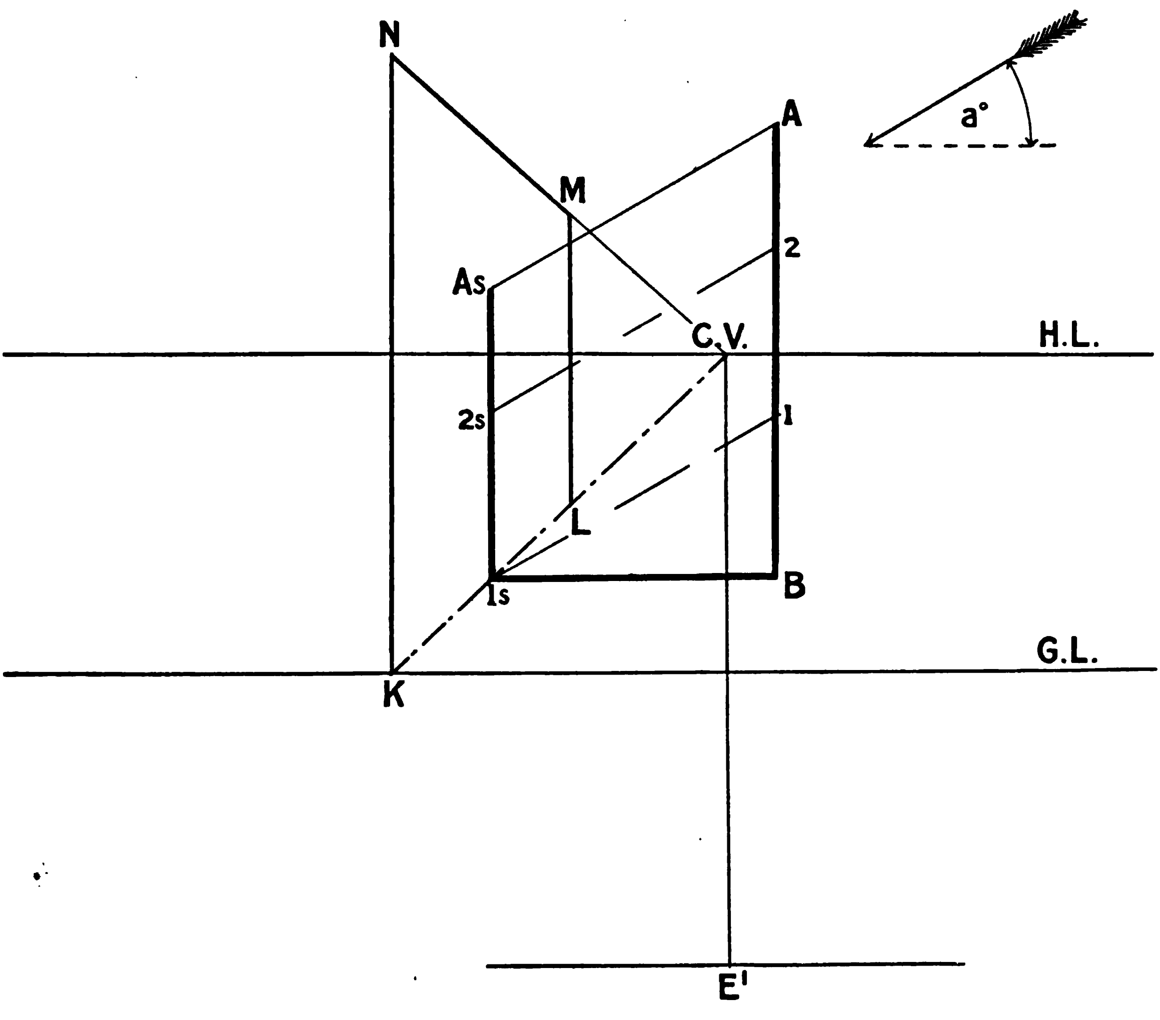
(*a*) PROBLEM X.

S shows the sun to be behind the **P.P.** on the spectator's right; its rays coming in vertical planes making **b°** with the **P.P.**, its altitude being **a°**.

If a vertical plane is supposed to contain the sun and **A B**, **V B** 1_S would be its intersection with the ground; a vertical line $1_S A_S$ would be its intersection with **K L M N**. The ray from **S** through **A** determines the shadow of **A** at A_S.

To find the point in **A B** which casts its shadow at 1_S, join 1_S**S** cutting **A B** at the required point **1**.

2 casts its shadow at 2_S, 2_S**2** being a ray (Rule 3).



(*a*) PROBLEM XI.

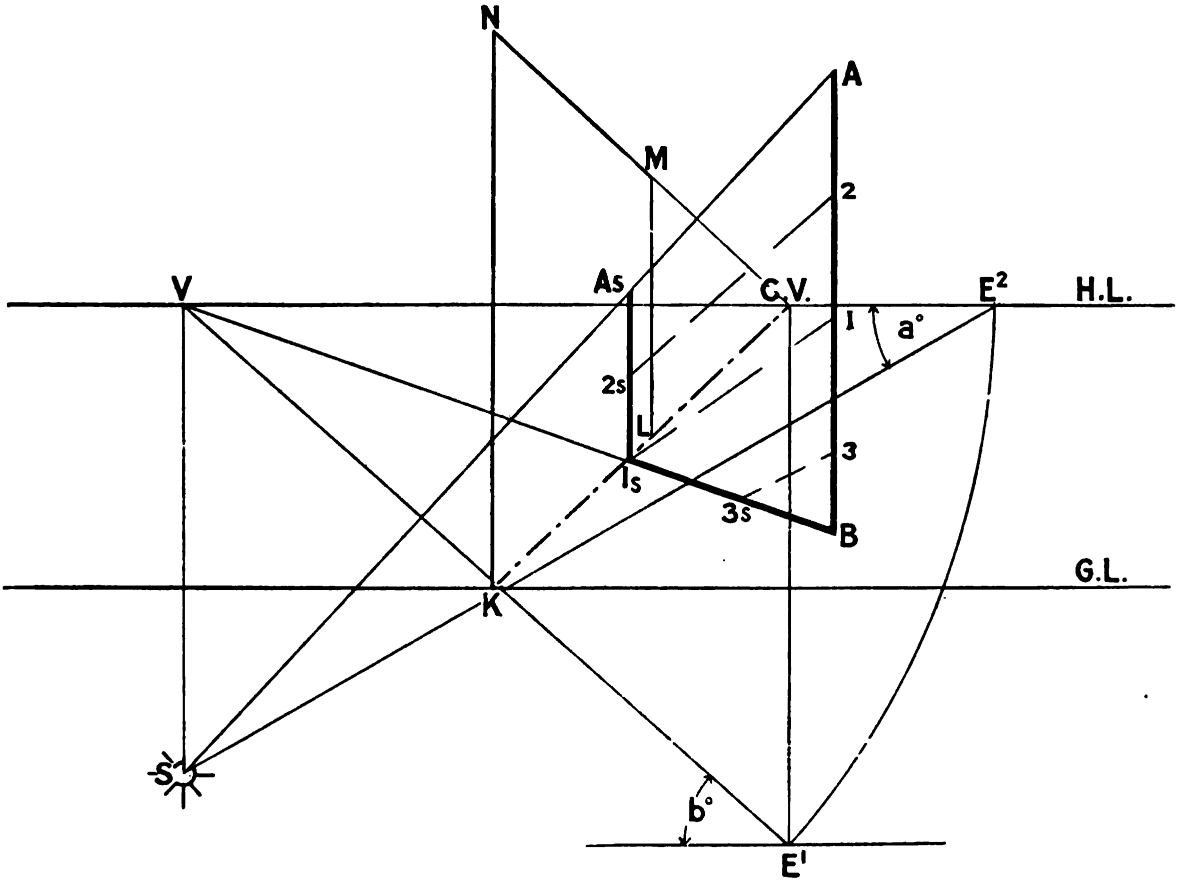
The arrow indicates that the sun is in the **P.P.** on the right at an altitude of **a**°.

A vertical plane containing the sun cuts the ground along **B** 1_S (a line parallel to the **G.L.**) and **K L M N** in a vertical line $1_S A_S$.

The ray through **A** (parallel to the arrow) determines the shadow of **A** at A_S.

1_S on **K L** is cast by **1**, which is found by drawing back the ray $1_S 1$ parallel to the arrow.

2 casts its shadow at 2_S.



(*a*) PROBLEM XII.

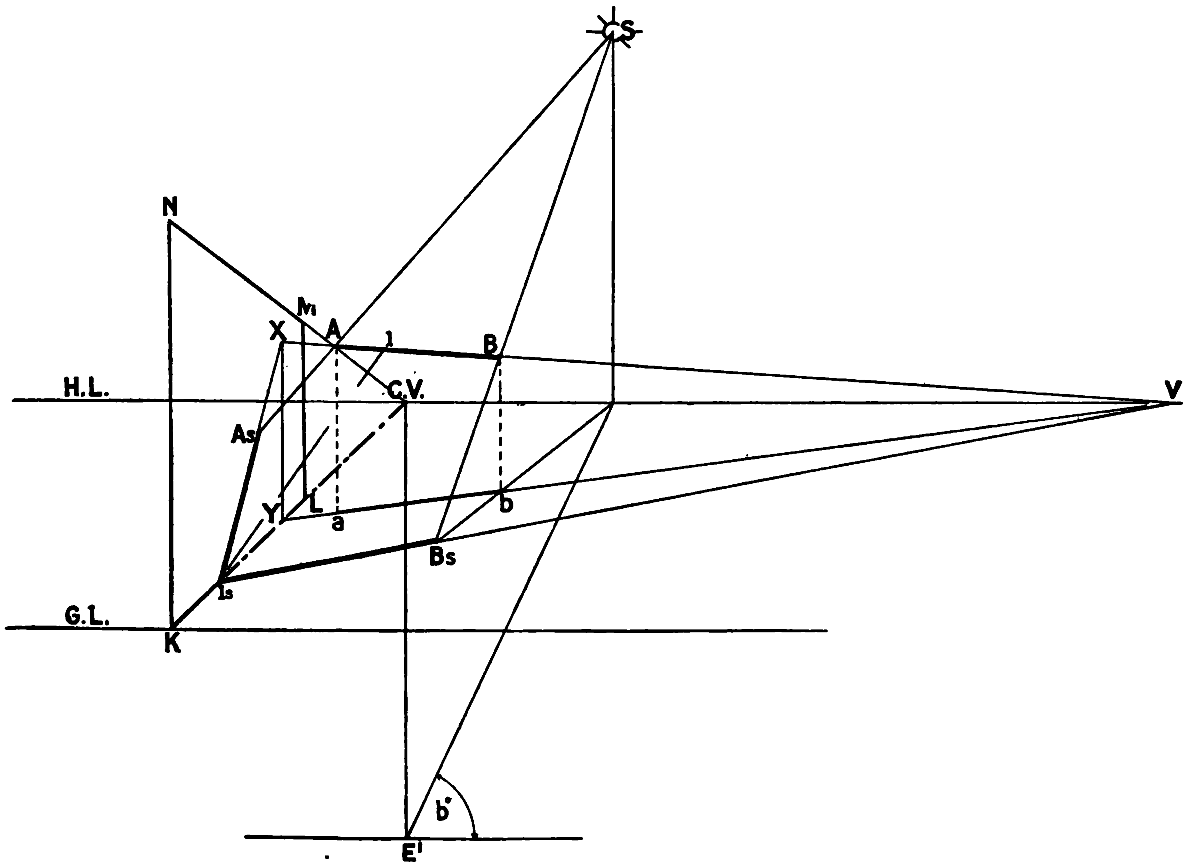
S shows the sun to be in front of the **P.P.** on the right; its rays coming in vertical planes making **b°** with the **P.P.**, its altitude being **a°**.

A vertical plane containing the sun intersects the ground in **B** 1_S**V** and **K L M N** in a vertical line $1_S A_S$. The ray from **S** through **A** determines the shadow of **A** at A_S.

To find the point in **A B** which casts its shadow at 1_S, produce **S** 1_S to cut **A B** at **1**.

In Problems XIII., XIV., and XV., **K L M N** is a vertical plane. **A B** is a horizontal line, vanishing at **V** (**a b** being the plan of **A B**).

The sun is situated in similar positions to that used in the preceding problems.



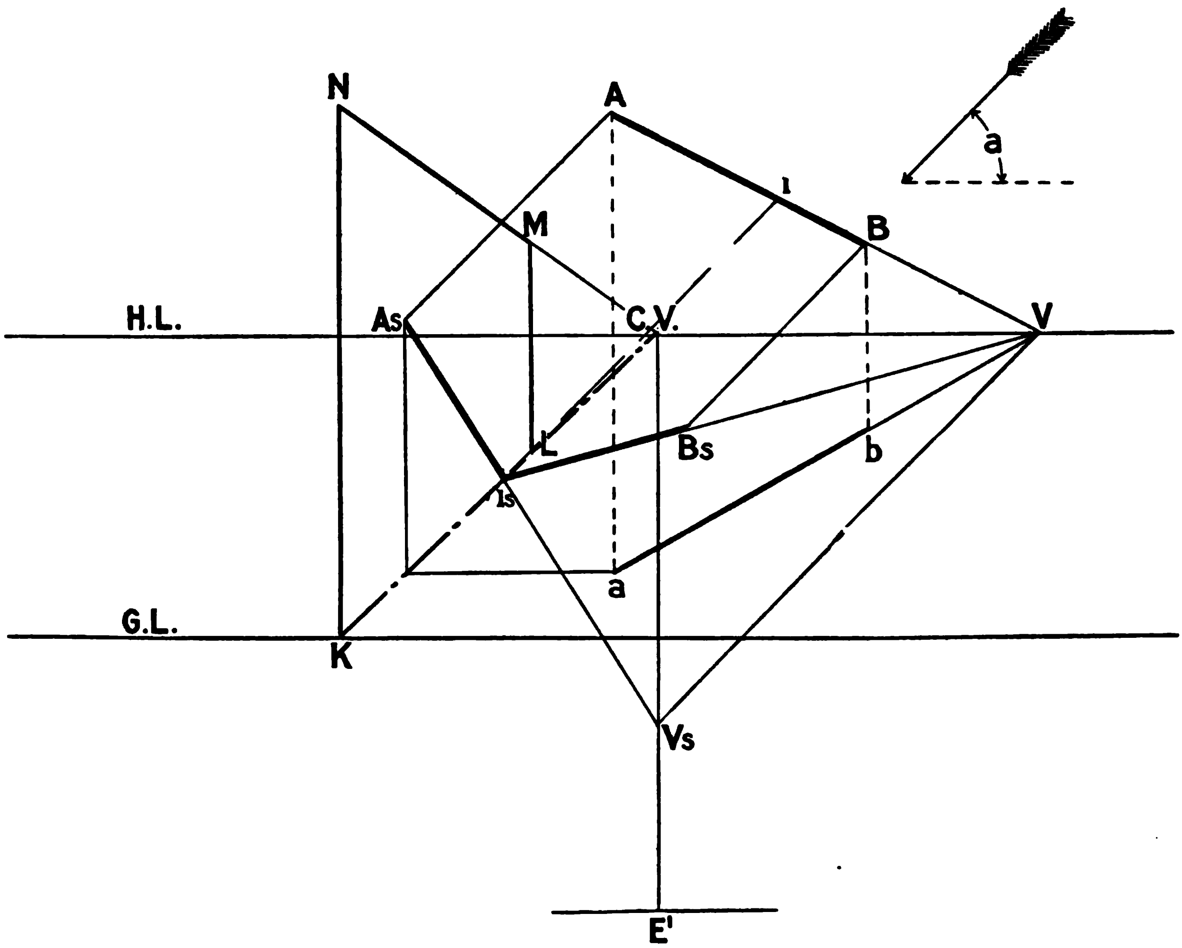
(*b*) PROBLEM XIII.

b a produced cuts **K L** at **Y, X** is the intersection of **B A** with **K L M N,** obtained by drawing a vertical line **Y X** cutting **B A** produced.

B_S is the shadow of **B** obtained by finding the shadow of **B b.** **V** B_S is produced to cut **K L** at 1_S. 1_S is joined to **X,** and **S A** is produced to cut 1_S**X** at A_S.

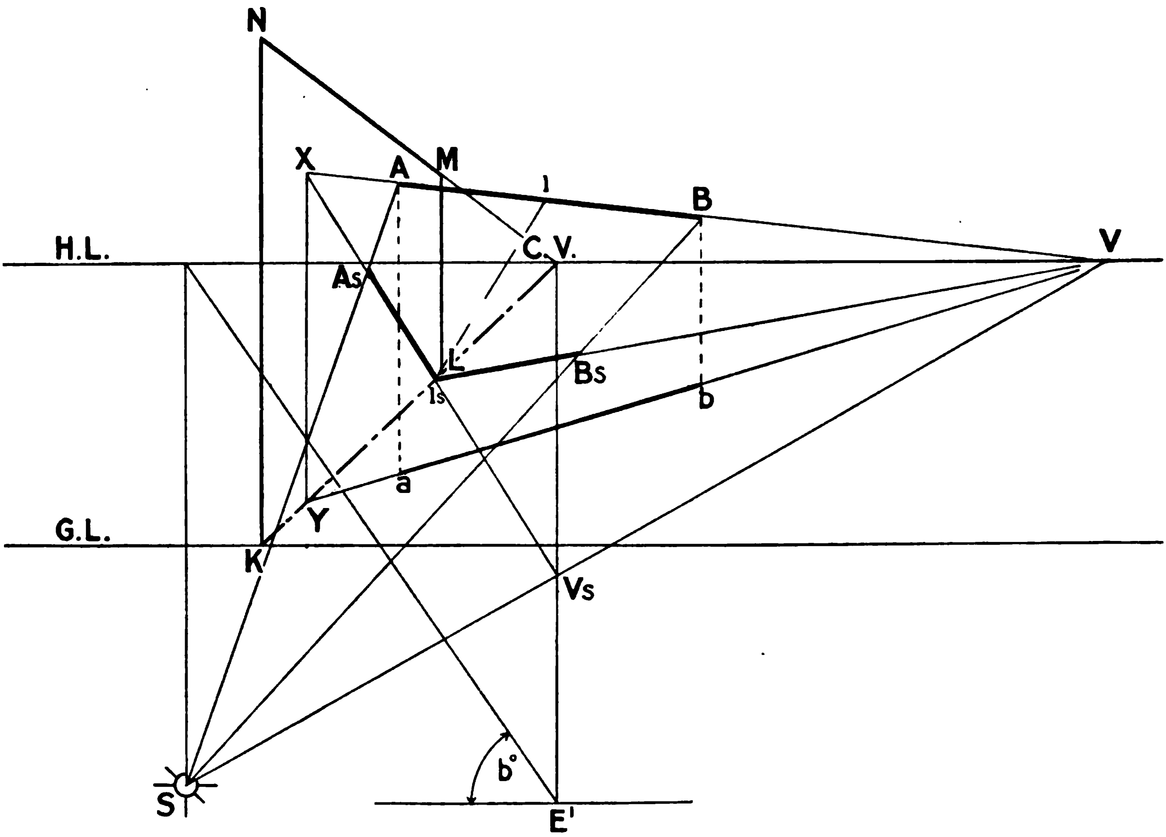
B_S1_S is a portion of **A B**'s shadow on the ground (Rule 5). 1_SA_S is the shadow of **A B** on **K L M N** (Rule 4).

1_S**S** cuts **A B** at **1,** the point which casts its shadow on **K L.**



(*b*) PROBLEM XIV.

A_S is obtained by finding **A a**'s shadow. Find the **V.P.** of the shadow cast by **A B** on **K L M N**. **V** V_S is parallel to the ray and cuts **C.V.** E^1 at V_S (Rule 6*a*). A_SV_S is joined, cutting **K L** at 1_S. 1_SB_S vanishes at **V** (Rule 5). **B** B_S is parallel to the ray.

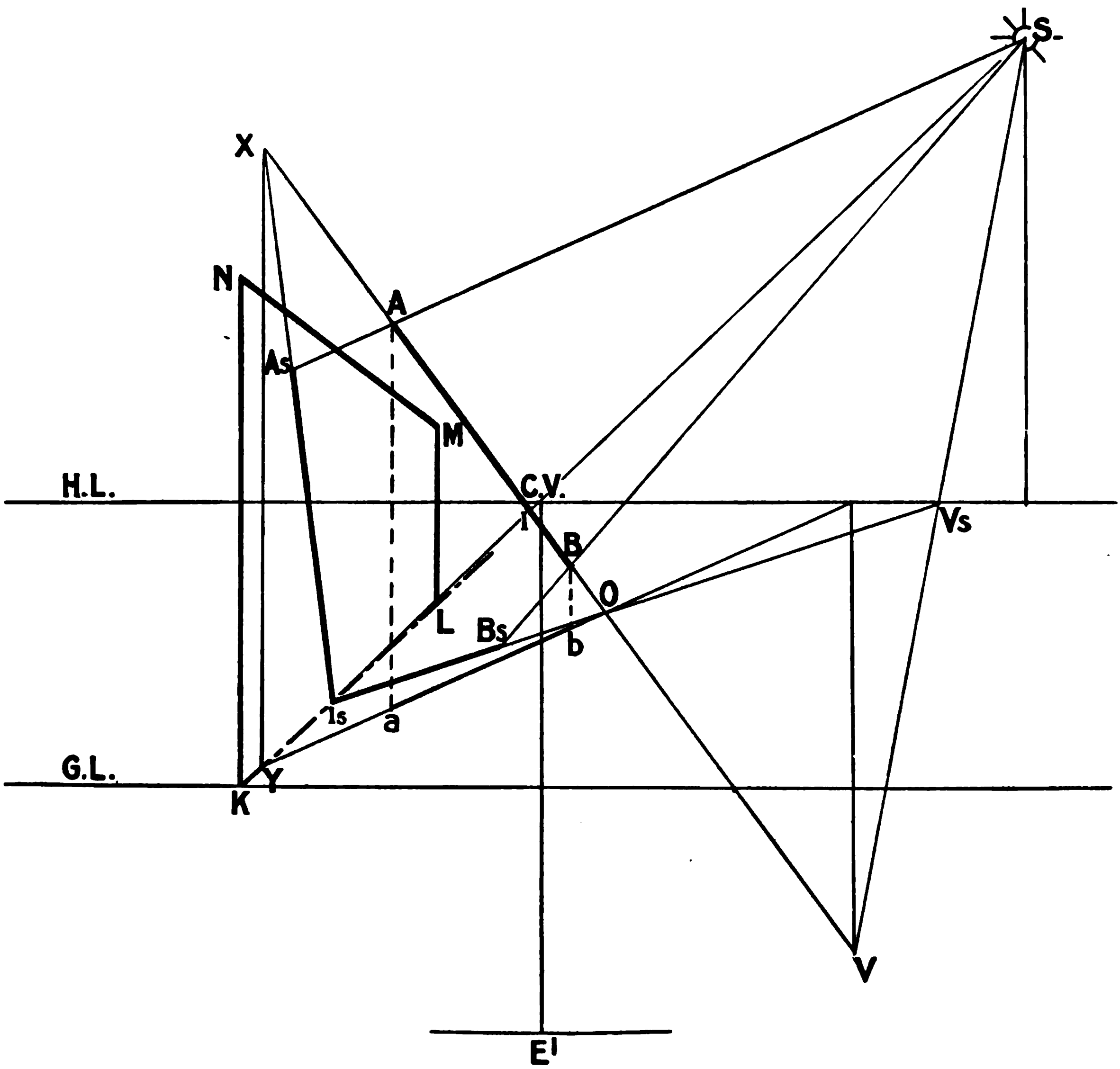


(*b*) PROBLEM XV.

A B cuts **K L M N** at **X**. **A B**'s shadow vanishes at V_S; obtained by joining **V S**, which cuts **C.V.E**[1] at V_S (Rule 6*a*). **X** V_S cuts **K L** at 1_S. 1_S is joined to **V** (Rule 5). A_S and B_S are obtained by drawing the rays **A S** and **B S**.

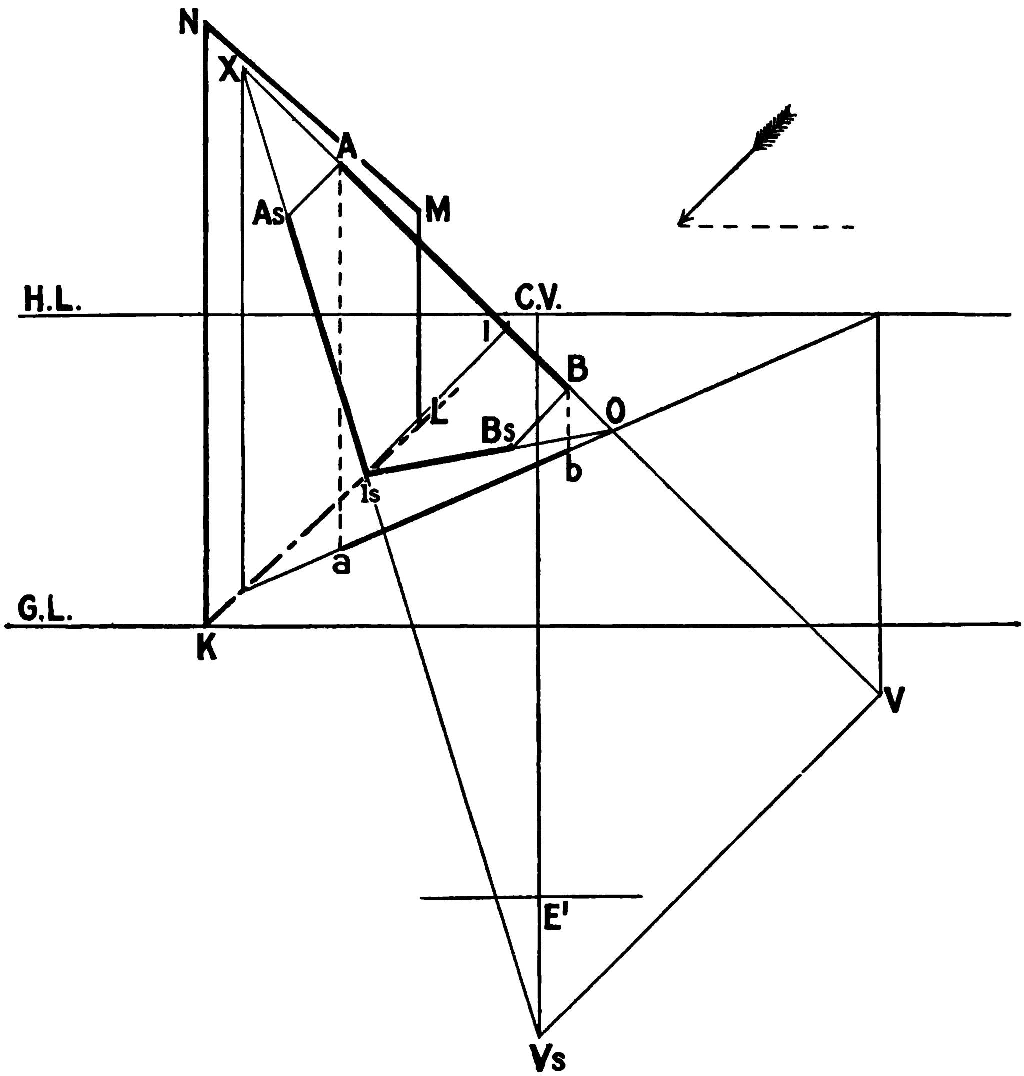
In Problems XVI., XVII., and XVIII., **K L M N** is a vertical plane; **a b** is the plan of an oblique line **A B** which vanishes at **V**.

The sun is situated in similar positions to that used in the preceding figures.



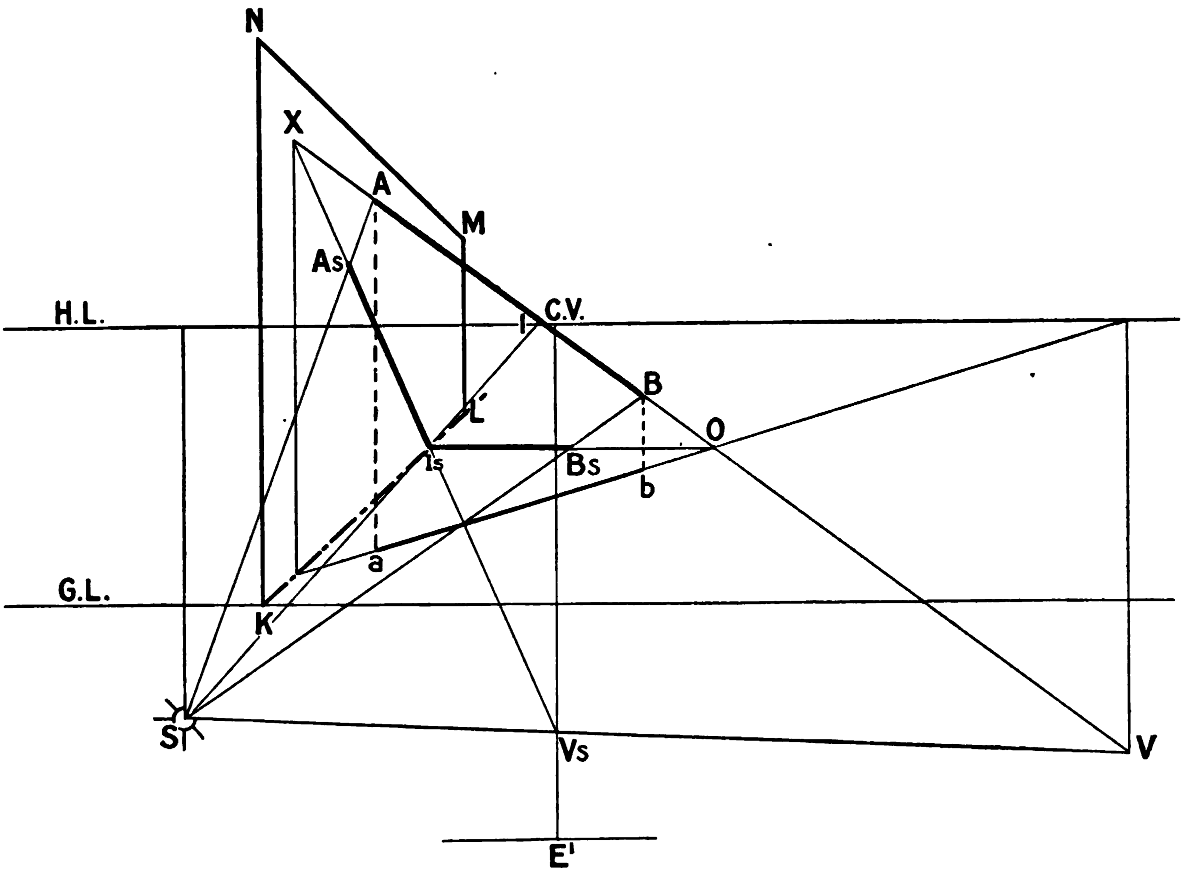
(*c*) PROBLEM XVI.

b a produced cuts **K L** at **Y**. **X** is the intersection of **B A** with **K L M N**, obtained by drawing a vertical line **Y X** cutting **B A** produced. **O** is the intersection of **A B** with the ground, obtained by producing **A B** to cut **a b** produced. The shadow of **A B** on the ground passes through **O** and vanishes at V_S, obtained by joining **V S**, which cuts **H.L.** at V_S (Rules 5 and 6*a*) $V_S O$ produced cuts **K L** at 1_S. $X 1_S$ is joined to obtain the shadow of **A B** on **K L M N** (Rule 5). **S A** and **S B** produced determine A_S and B_S.



(*c*) PROBLEM XVII.

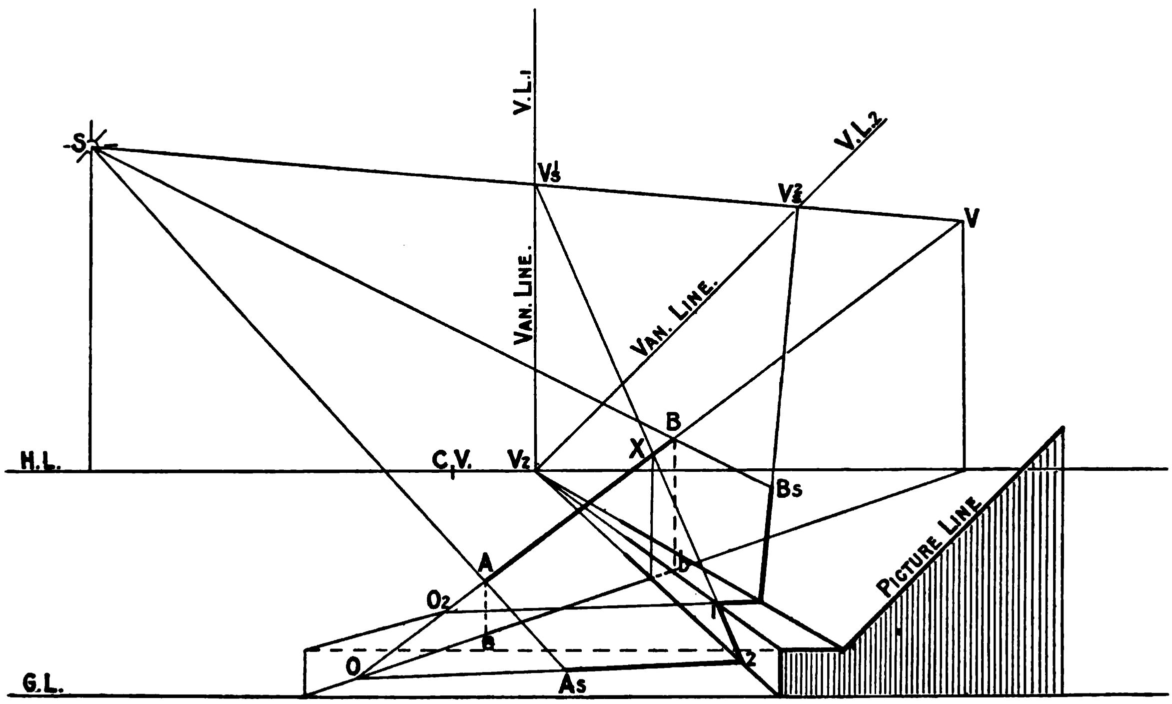
A B cuts the ground at **O** and **K L M N** at **X**. **V V**$_S$ is drawn parallel to the given ray. The shadow of **A B** on **K L M N** passes through **X** and vanishes at **V**$_S$ (Rules 5 and 6*a*). **X V**$_S$ cuts **K L** at **1**$_S$. **1**$_S$ is joined to **O**. **A A**$_S$ and **B B**$_S$ are parallel to the given ray.



(*c*) PROBLEM XVIII.

A B cuts the ground at **O** and **K L M N** at **X**. The **V.P.** of **A B**'s shadow on **K L M N** is obtained by joining **V S** cutting **C.V. E**[1] at V_S (Rule 6*a*). **X** is joined to V_S and cuts **K L** at 1_S. 1_S is joined to **O** (Rule 4). A_S and **B** are found by joining **A S** and **B S**.

(*d*) In Problems XIX., XX., and XXI. a terrace composed of an oblique plane, a horizontal plane, and a vertical plane, and an oblique line **A B** (vanishing at **V**) are shown.



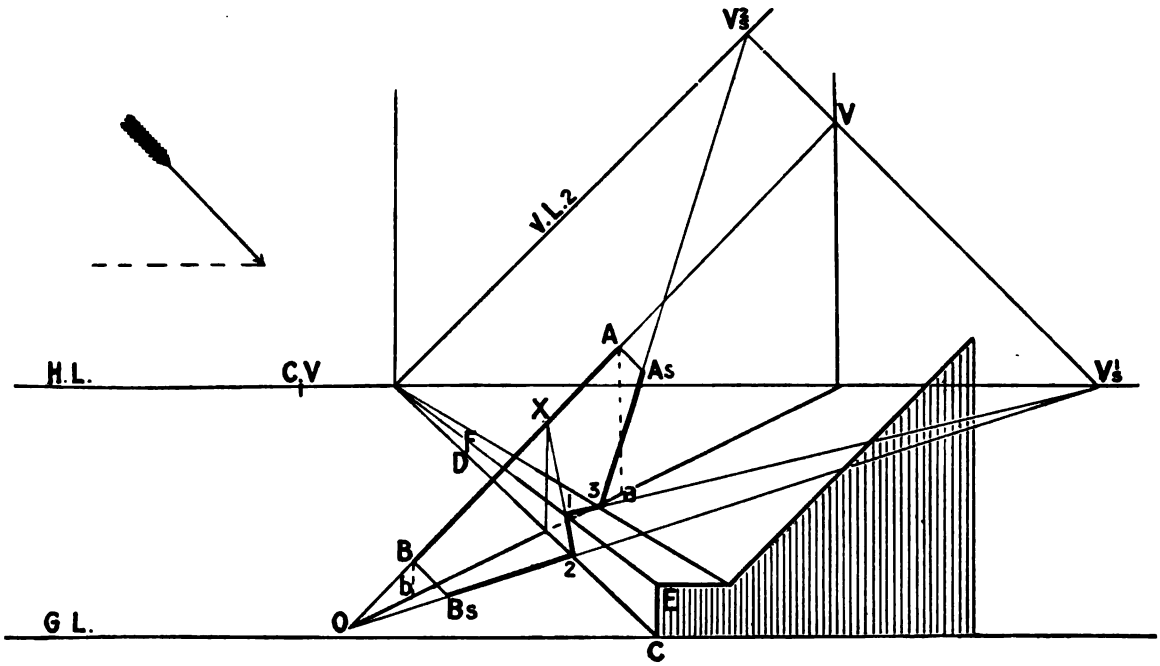
(*d*) PROBLEM XIX.

The sun is situated behind the **P.P.**, on the spectator's left, the **V.P.** of its rays being at **S.**

A B cuts the ground at **O**, the plane containing the horizontal part of the terrace at $\mathbf{O_2}$ (as $\mathbf{O_2}$ is in **A B** at the same height as the horizontal part of the terrace is above the ground), and the vertical plane at **X**.

The shadow of **A B** on the vertical part of terrace passes through **X** (Rule 4), and vanishes at $\mathbf{V_S^1}$ (Rule 6*a*). This determines **1** and **2**.

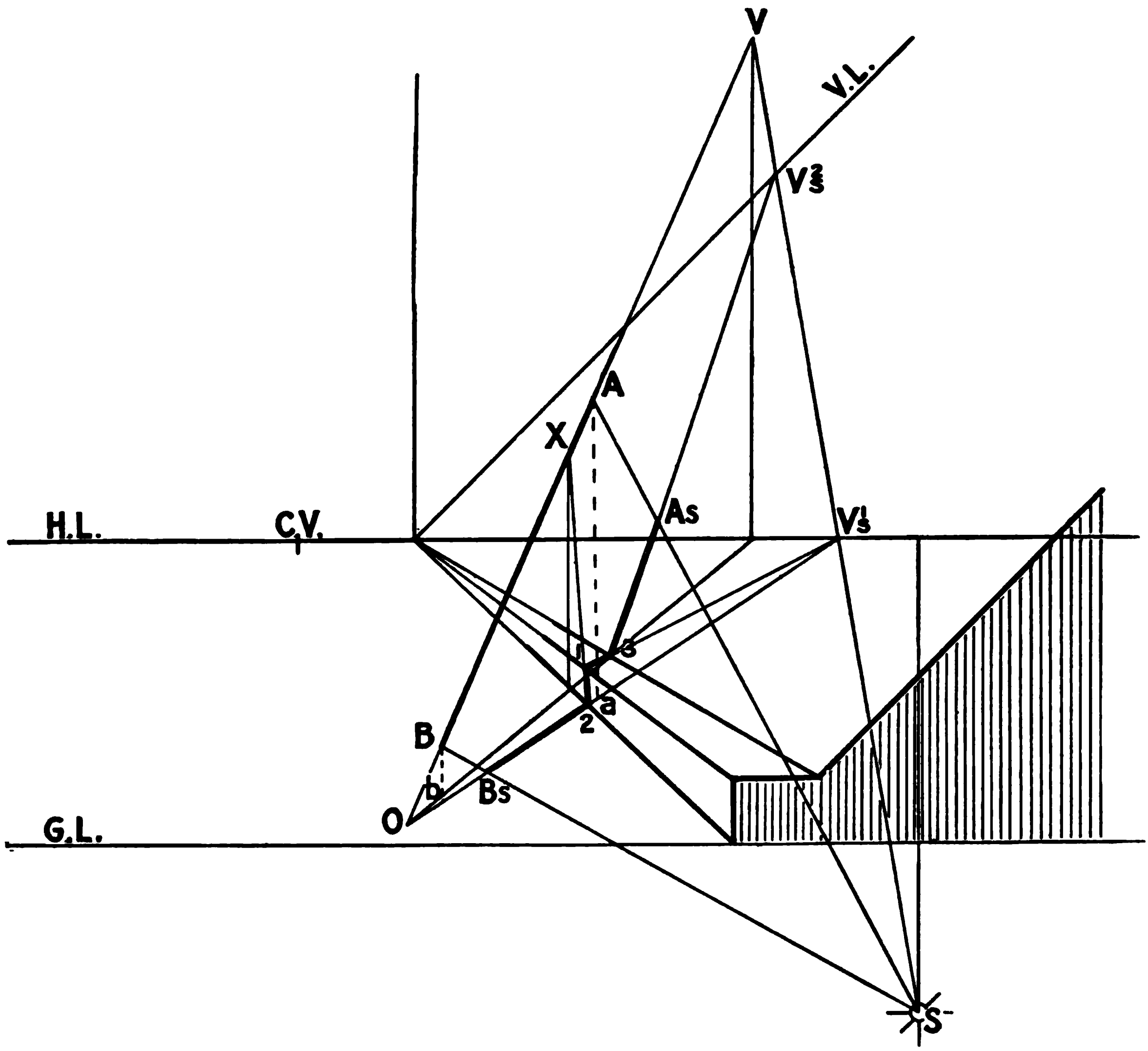
The shadow on the ground begins at **2** and runs towards **O** (Rule 4); **As** being obtained from the ray **S A** (Rule 3). On the horizontal part of the terrace the shadow runs in the direction $\mathbf{O_2 1}$ (Rule 4), and then runs on the oblique part of terrace, vanishing at $\mathbf{V_S^2}$ (Rule 6*a*).



(*d*) PROBLEM XX.

The sun is in the **P.P.** at the altitude shown.

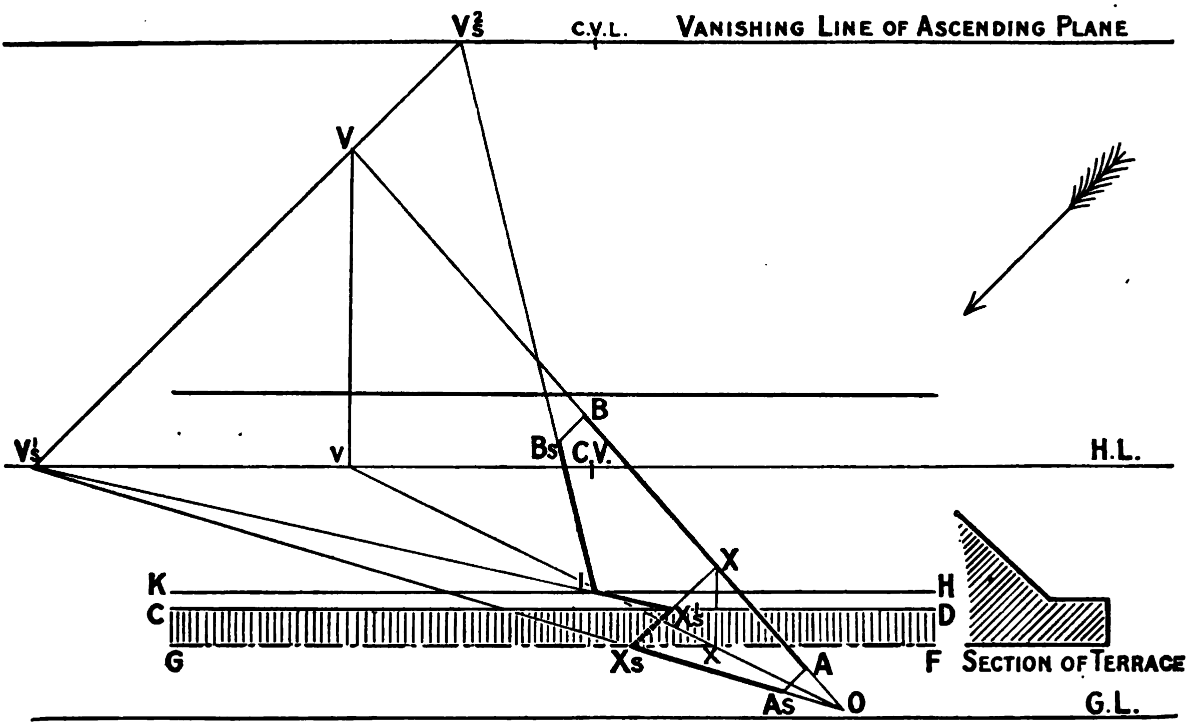
X and **O** correspond to **X** and **O** in the preceding figure (Prob. XIX.). V^1_S is obtained by drawing $V V^1_S$ parallel to the given ray (Rule 6*a*). $O V^1_S$ is the shadow of **A B** on the ground (Rule 4); this line cuts **C D** at **2**; the shadow then runs towards **X**, cutting **E F** at **1** (Rule 4) on the horizontal part towards V^1_S to **3**, and finally on the oblique part towards V^2_S (Rule 6*a*).



(*d*) PROBLEM XXI.

The sun is situated in front of the **P.P.**, on the spectator's left, the **V.P.** of its rays being at **S**.

The shadow of **A B** on the ground passes through **O** (Rule 4) and vanishes at V_s^1 (Rule 6*a*). At **2** the shadow runs towards **X** to point **1** (Rule 4), then towards V_s^1 to **3** (Rule 6), and finally from **3** towards V_s^2 (Rule 6).



(*e*) PROBLEM XXII.

A terrace composed of an ascending plane, a horizontal plane, and a plane parallel to **P.P.**, and an oblique line **A B** (vanishing at **V**), are shown.

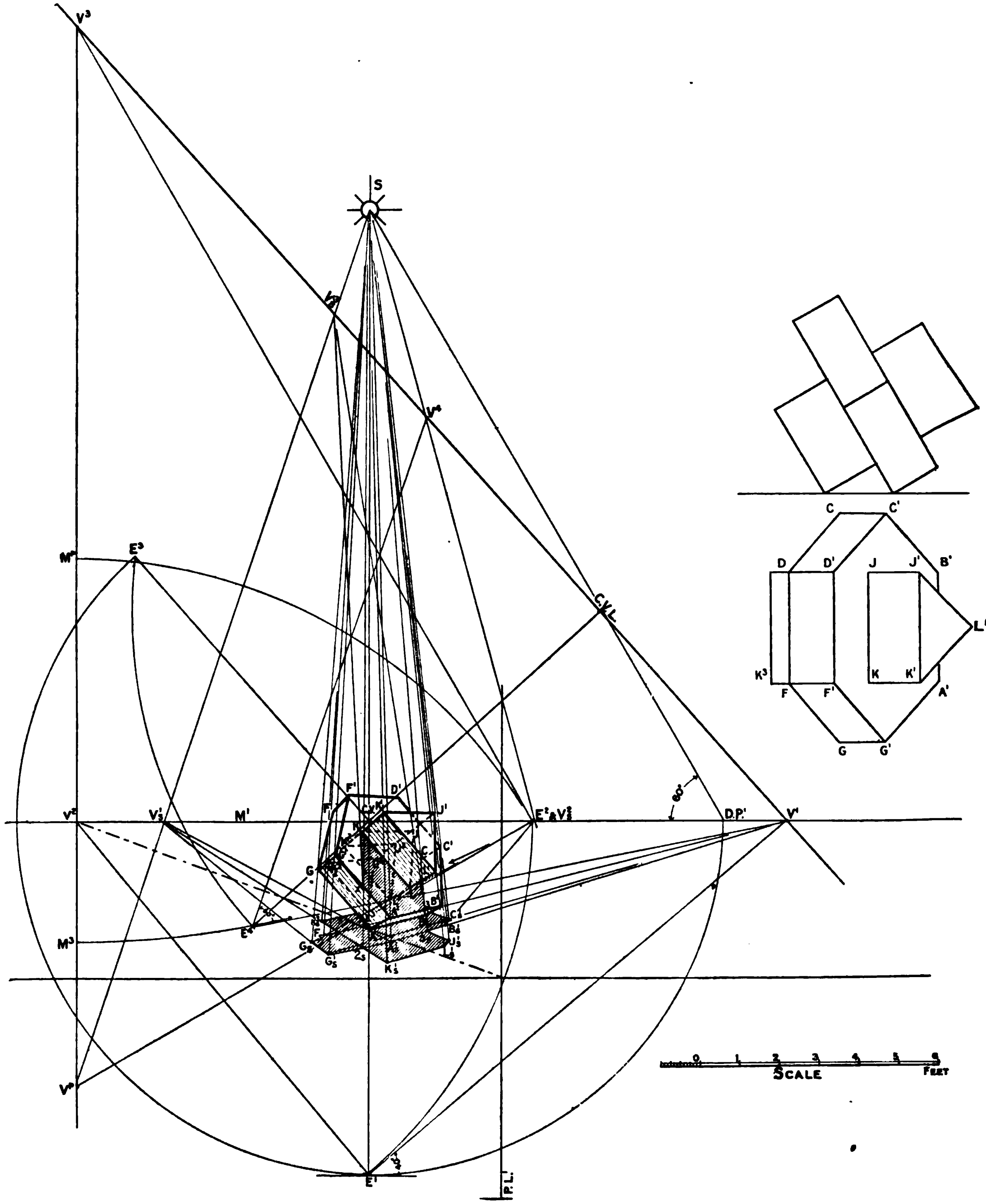
The sun is situated in the **P.P.** at the altitude shown.

O is the intersection of **B A** with the ground, and **X** its intersection with the vertical plane **C D F G**.

The shadow of **A B** on the ground runs from **O** (Rule 4) towards V^1_S (Rule 6).

The sun is in the same plane as **C D F G** and hence that surface is in shadow (**C D**'s shadow coinciding with **F G**); notice that **X** casts its shadow at X_S and also at X^1_S; the shadow then runs on **C D H K** from X^1_S towards V^1_S to **1**, then on the ascending plane towards V^2_S (Rule 6*a*).

PROBLEM XXIII.



PROBLEM XXIII.

(FROM A RECENT EXAMINATION PAPER.)

(*a*) *The scale to be used in working this problem is* $\frac{1}{2}$ *in. to* 1 *ft. The diagram is drawn to scale and is the plan and elevation of a compound solid; put this into perspective (using an oblique vanishing line) in the same position as the diagram, but having the edge of the hexagon which rests on the ground, vanishing towards the right at an angle of* 40° *to the picture plane, starting from a point* 4 *ft. below the eye, opposite the centre, and* 4 *ft. within the picture, and the sides of the prism sloping downwards towards the left.*

(*b*) *Show the shadows cast by the solid having the light from within the picture inclined at* 60° *to ground in planes perpendicular to the picture.*

The distance to the spectator is 9 *ft.*

(*a*) V^3V^1 is the vanishing line of the hexagonal faces.

A F, A^1F^1 and B^1D^1 vanish at V^3.

V^4 is the **V.P.** of G^1F^1, B^1C^1, L^1J^1, &c.; the vanishing parallel of G^1F^1, B^1C^1, L^1J^1, &c., makes 60° with E^4V^1, the vanishing parallel of the horizontal trace **A B**.

$P.L^1$. is the picture line of a vertical plane containing **F** F^1 **A** A^1 and K^1K^3, and is used for measuring on **A** A^1 and K^1K^3.

The complete construction lines are omitted, to avoid confusion, but with the aid of the finished drawing the perspective representation should present no difficulty.

(*b*) *Find* **S** *the* **V.P.** *of the sun's rays*:—The sun is within the picture, therefore *behind the* **P.P.**; as its rays come in planes perpendicular to the **P.P.**, the **V.P.** of these rays must lie in a vertical line through the **C.V.**

Rotate the eye into the **P.P.** about **C.V. S**, this gives $D.P^1$.; set off the sun's altitude, *i.e.* 60°, and thus determine **S** on **C.V. S.** Begin at **A**; find the shadow cast by **A** A^1 on the ground; it will pass through **A** (**A** is on the ground) and it will vanish at V^1_S (obtained by Rule 6*a*, *i.e.* by joining V^P to **S** cutting the **H.L.** at V^1_S). Obtain A^1_S by drawing the ray **S** A^1.

A^1_S B^1_S vanishes at V^1 (Rule 5). B^1_S is found by drawing the ray **S** B^1.

B^1_S C^1_S vanishes at V^2_S (Rule 6*a*).

C C^1 would then cast the shadow C^1_S V^1_S, as **C** C^1 vanishes at V^P.

C^1G^1 is parallel to **A B**, therefore C^1_S G^1_S vanishes at V^1; determine G^1_S by drawing the ray through G^1; join A^1_S G^1_S.

G^1_S G_S vanishes at V^1_S (Rule 6*a*).

G_S F_S vanishes at V^2_S, and the shadow of **F D** vanishes at V^1.

This completes the shadow cast by the hexagon.

Find the shadow cast by the triangular prism on the face of the hexagon.

The shadows of **K** K^1 and **J** J^1 run from **K** and **J** respectively towards V^3_S. (Rules 4 and 6*a*) when these shadows reach the side of the hexagonal prism at **2** and **3**, they are then cast on the ground.

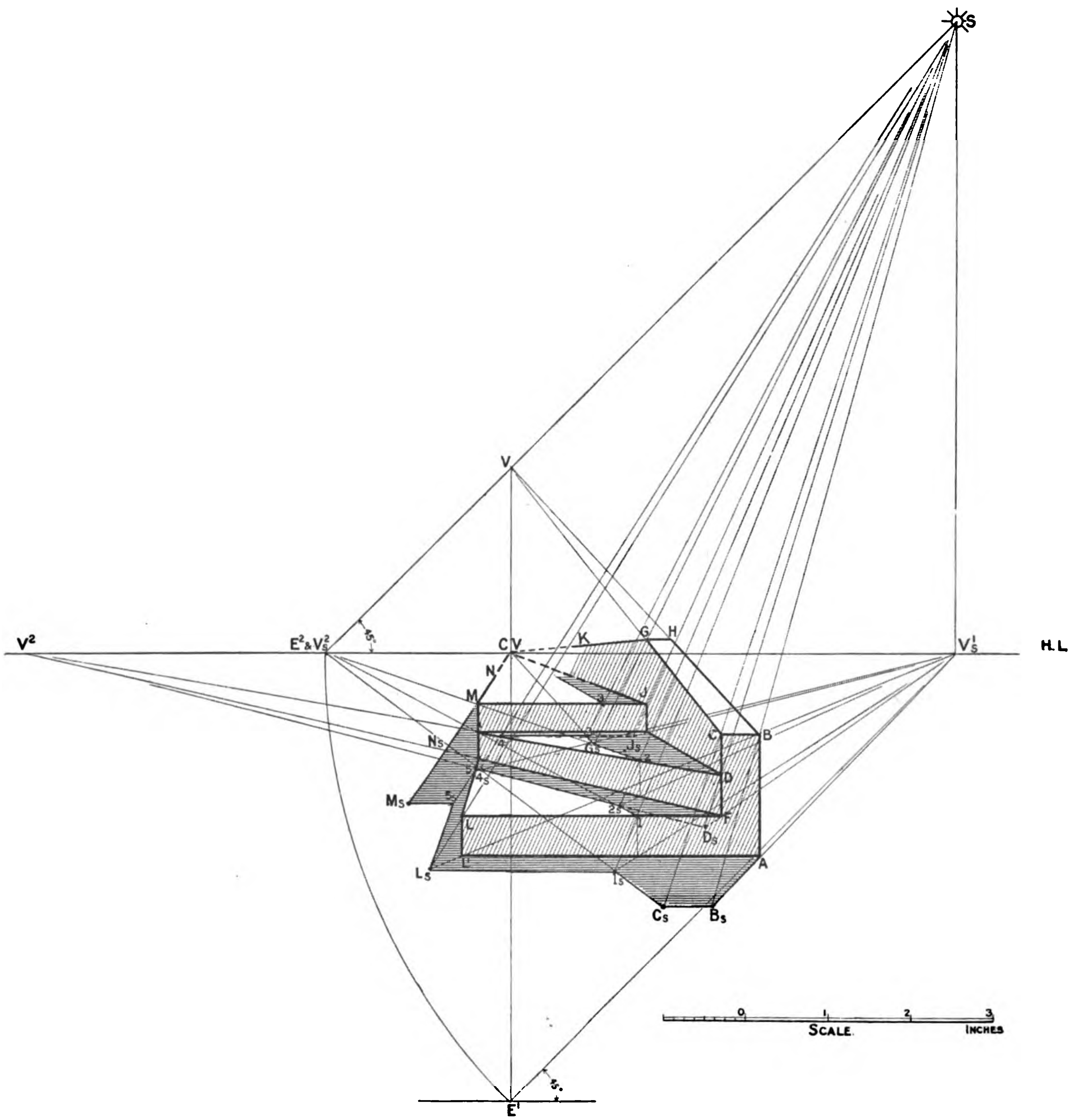
2 and **3** cast shadows at 2_S and 3_S (Rule 3).

3_S J^1_S vanishes at V^1_S, J^1_S L^1_S at V^2_S, and K^1_S 2_S K^3_S at V^1_S; join K^1_S L^1_S. The shadow of K^3 J^3 vanishes at V^1.

This completes the visible portion of the shadow.

Special attention should be paid to discover which lines of an object cast shadows and which parts of the object are in shadow. Try to realize the effect the sun would have when placed in the given position. Observe that lines such as **A** A^1 are known to have no effect on the shadow, as when A_S A^1_S is found it appears in the midst of another shadow.

PROBLEM XXIV.



PROBLEM XXIV.

Problem XXIV is the perspective representation of three steps and a dwarf parapet wall. The distance from the eye to the **P.P.** *is* 5 *ft.* 6 *ins. Cast the complete shadows upon the steps and the ground, the sun being in front of the spectator on his right at an altitude of* 45°, *its rays lying in vertical planes, making angles of* 45° *with the picture.*

Find **S** *the* **V.P.** *of the sun's rays as shown:*—**V** is the **V.P.** of **B H** and **C G**. V^2 is the **V.P.** of **D 4**.

AB_S vanishes at V^1_S (Rule 6*a*).

B_SC_S is horizontal.

C_S1_S vanishes at V^2_S (Rule 6*a*). V^2_S coincides with E^2.

Find the shadow of **L** *on the ground:*—L^1L_S vanishes at V^1_S. L_S1_S is horizontal and cuts C_S1_S at 1_S. 1_S is cast by **1** on the edge **F L**.

The shadow of **C G** on the surface **F L 5** vanishes at V^2_S.

D_S is the position of the shadow of **D** on the surface **5 L F** produced. D_S4_S vanishes at V^2.

D_S4_S cuts $1{,}2_S$ at 2_S.

2_S is cast by **2** on **D 4**.

The shadow of **C G** on the second step vanishes at V^2_S and is terminated by the ray through **G** at G_S. **G K** next casts a shadow on the same step; this shadow begins at G_S and vanishes at the **C.V.**

J_S is the shadow of **J** on the top surface of the second step.

J_S4, a level line, is the shadow of the edge **J M** on that step.

J_S4 cuts G_S **C.V.** at 3_S. 3_S is cast by **3** on **J M**.

The shadow of **G K** on the top step vanishes at the **C.V.**

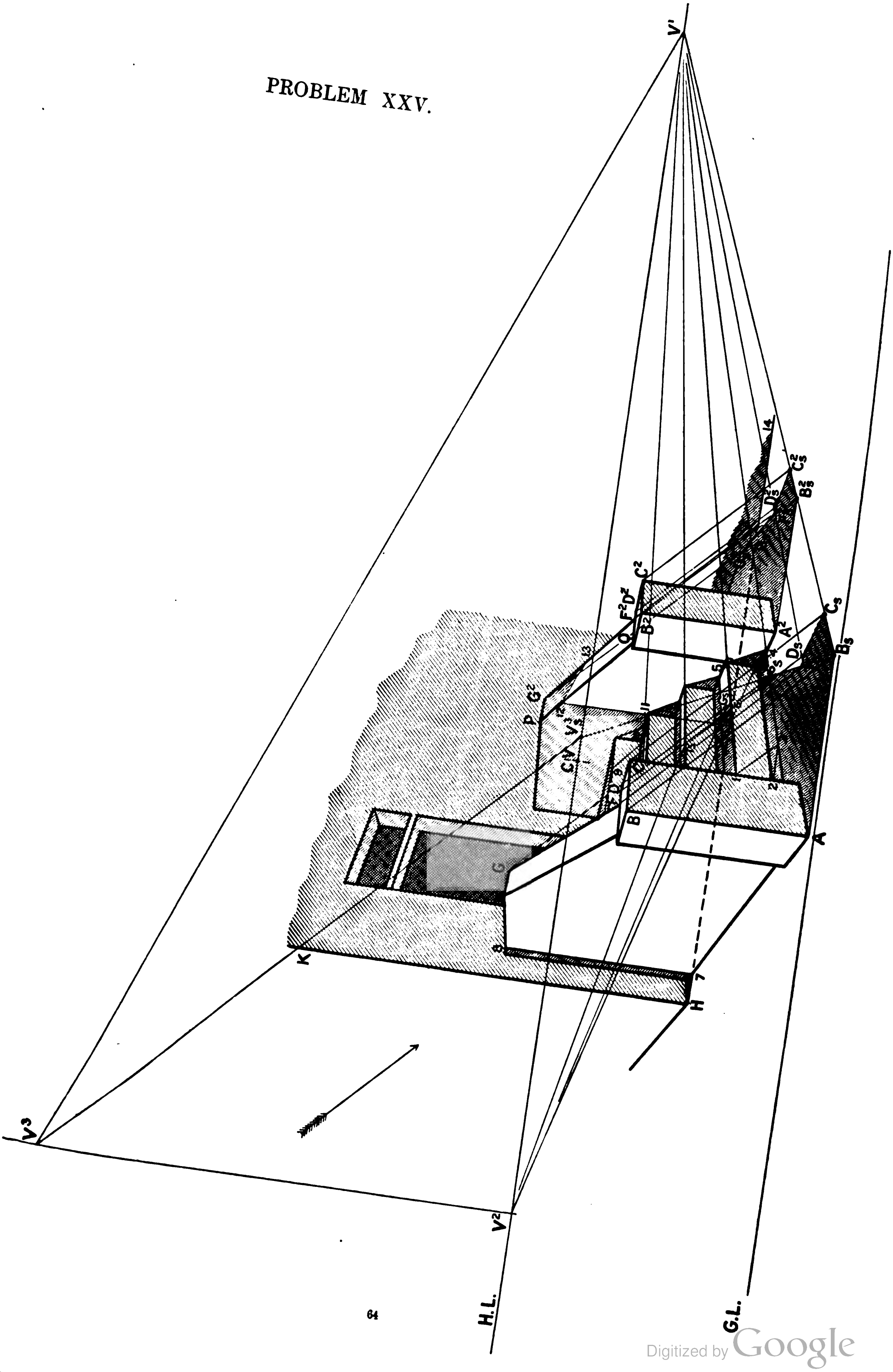
The shadow of **J M** on the second step falls off the second step at **4** on the edge **D 4**. The shadow of **4** is at 4_S on D_S4_S. **J M** then casts 4_S5 on the bottom step. 4_S5 is level.

L 5 casts its shadow on the ground at L_S5_S, which vanishes at the **C.V.**

Part of **J M** casts 5_SM_S on the ground.

M_SN_S vanishes at the **C.V.**

PROBLEM XXV.



PROBLEM XXV.

(From a Recent Examination Paper.)

The diagram is the perspective representation of a doorway with steps. Show the shadows cast when the sun is parallel to the picture plane, and inclined at 45° to the horizon downwards towards the right hand.

A B_S is parallel to the **P.P.** B_S being obtained by drawing the ray **B** B_S at 45° to the **H.L.**

$A^2B^2_S$ is also horizontal, B^2_S being on B_SV^1.

C_S and C^2_S lie on B_SV^1 and are obtained by drawing the rays through **C** and C^2.

C_SD_S and $C^2_SD^2_S$ vanish at V^2; draw the rays through **D** and D^2. F_SD_S and $F^2_SD^2_S$ lie on V^1D_S and on the rays.

V^3_S is obtained by drawing $V^3V^3_S$ to intersect the **H.L.** at V^3_S (Rule 6*a*). The shadows of **F G** and F^2G^2 vanish at V^3_S.

Find the shadows cast by the steps.

1,2 is any vertical line on the upright portion of the lowest step. **3** is the shadow of **1** on the ground.

The shadow of the edge **1,5** passes through **3** and vanishes at V^1 This determines **4**, the shadow then runs to **5** on the vertical surface.

In a similar way find the shadow cast by each step.

Find the shadow cast by **K H**.

As **K H** is a vertical line, its shadow, **H 7**, 13_S, **14** is horizontal. **7,8** is vertical as it is on a vertical surface. 13_S on $F^2_SG^2_S$ is the shadow cast by the point **13** situated on F^2G^2. V^3V^1 is the **V.L.** of the surface **P** G^2F^2Q; the shadow of **K H** on **P** G^2F^2Q is **13,12** parallel to V^3V^1.

12,11 is vertical; **11,10** is horizontal and meets the shadow cast by the upper edge of the top step, on the second step at **10**. **10** is cast by **9** on the top step.

A horizontal line through **9** completes the shadow cast by **H K**.

The face of the door is in shadow.

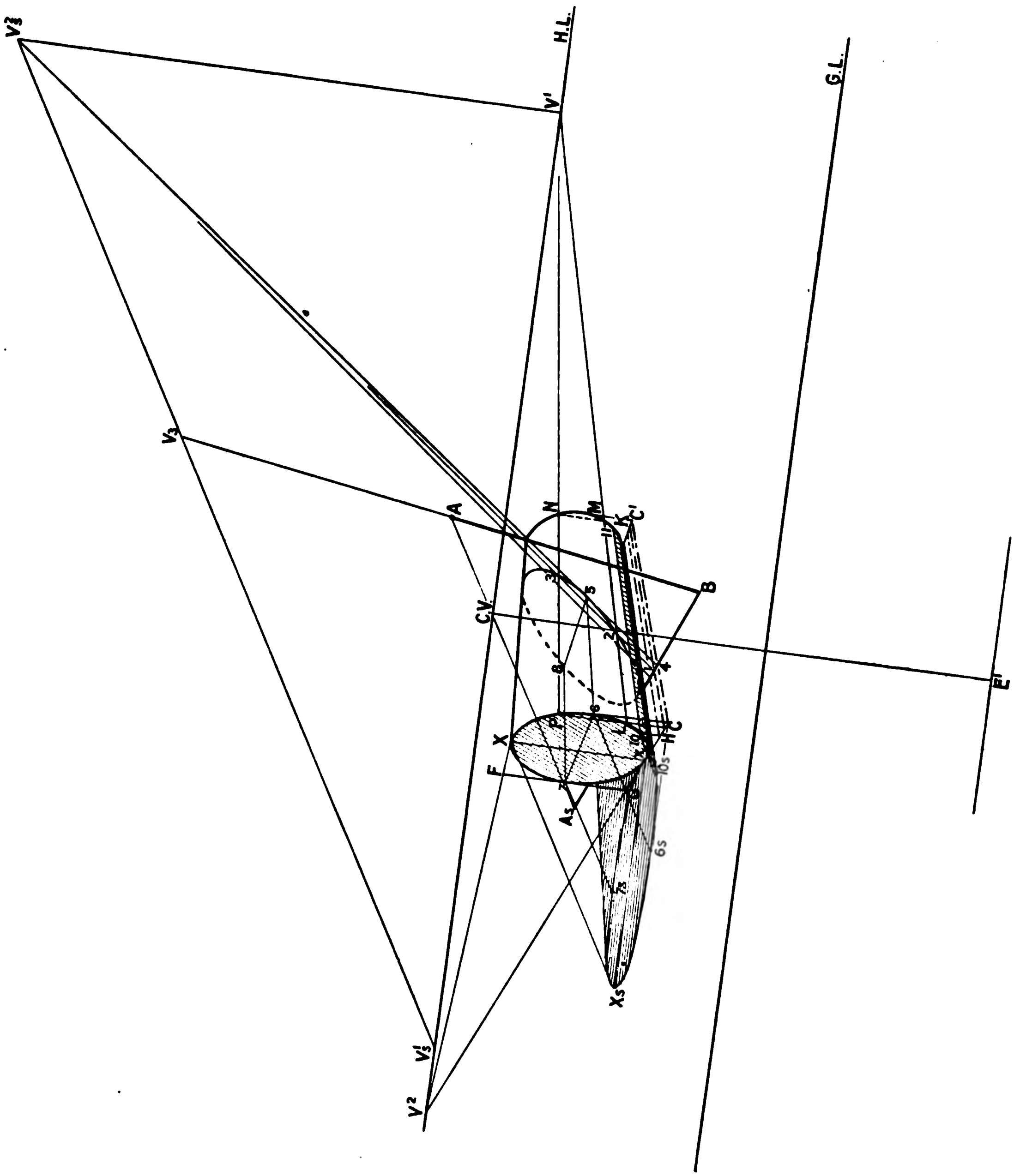
The shadow of **F G** cuts the shadow of the bottom step at 6_S; 6_S is cast by **6** on the edge **1,5**.

6 G_S vanishes at V^3_S and is terminated by the ray.

The horizontal line through **G** and vanishing at V^2 casts a shadow G_S15_S vanishing at V^2.

15_S is cast by **15**; the shadow on the next step vanishes at V^2.

PROBLEM XXVI*.



PROBLEM XXVI*.

To find (*a*) *the shadow cast by a cylinder on the ground.*
(*b*) *the shade lines of a cylinder.*
(*c*) *the shadow of a line on a cylinder.*

The above figure shows the perspective representation of a cylinder lying on the ground with its axis horizontal. The sun is situated in the plane of the picture on the spectator's right, its rays making 30° with the ground.

(*a*) **G C** is the lower edge of a square surrounding the circle; **G C** lying on the ground plane.

Take a point **X** on the curve. Find **x** on the ground below **X**. Obtain the shadow of the vertical line **X x**, *i.e.* **x** X_S; the ray through **X** gives the shadow of **X** at X_S.

In a similar way obtain the shadow of a series of points of the curve on the ground.

Draw a fair ellipse through the points thus obtained, and **x**, 7_S, X_S, 6_S, 10_S, the shadow of one end on the ground, will be determined.

Proceed in a similar manner with the other end.

The shade lines on the cylinder must be parallel to its axis and hence vanish at V^1; the shadows cast by the shade lines therefore vanish at V^1; these shadows are obtained by drawing the common tangents to the two ellipses, forming the shadows of the cylinder's ends, taking the two tangents which vanish at V^1.

(*b*) 10_S is the point in which the shadow cast by the shade line touches the shadow of the cylinder's end. 10_S is cast by **10** (determined by drawing the ray 10_S **10** to cut **X 6 x** at **10**).

10 is a point in the shade line. **10, 11**, the complete shade line, vanishes at V^1.

The other shade line is obtained in a similar way.

(*c*) **A B** is an oblique line, vanishing at V_3 and having the end **B** on the ground.

B V^1_S is the shadow of **A B** on the ground (Rule 6*a*).

H K is the horizontal trace of a vertical plane, parallel to the axis of the cylinder and cutting the cylinder at **L M** and **P N**, which vanish at V^1.

Note that **L M** and **P N** lie on the cylinder and on the plane.

$V^1 V^2_S$ is the **V.L.** of all vertical planes parallel to the cylinder's axis. V^2_S is the **V.P.** of **A B**'s shadow on these planes.

The shadow of **A B** on the ground cuts **H K** at **1**; suppose the shadow to run on the plane **P H K N**. **1** V^2_S is **A B**'s shadow on that plane.

Let **1** V^2_S cut **L M** and **P N** at **2** and **3**.

The points **2** and **3** are two points of **A B**'s shadow on the cylinder.

Proceed in a similar way to find other points.

N.B. **C** C^1 is the horizontal trace of a vertical plane touching the cylinder. **6,5** is this plane's line of contact with the cylinder. **4** is on **C** C^1. **4,5** vanishing at V^2_S is the shadow of **A B** on this plane, giving **5** as the shadow on the cylinder of a point in **A B**.

SHADOWS CAST BY AN ARTIFICIAL LIGHT.

The rays proceeding from an artificial source of illumination radiate from a point. In problems the "source" and its plan are given.

By referring to fig. 31 the method of obtaining the shadow of a vertical line will be easily understood: a vertical plane is supposed to contain the "source" (**L**) and the line **A B**; its intersection with the surfaces **K L M N** and **L O P M** determines the line's shadow.

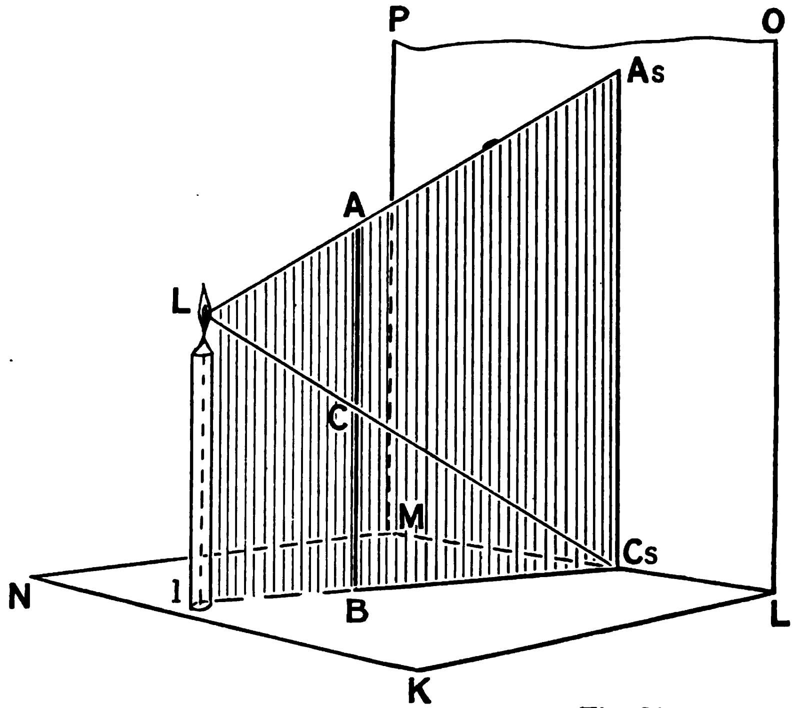


Fig. 31.

Many problems are worked by drawing vertical lines from the various points of the figure to the ground and obtaining the shadow of these vertical lines; however, a neater solution is often obtained by finding the **V.P.** of the shadow of the lines composing the figure.

TO FIND THE VANISHING POINT OF THE SHADOW OF A LINE.

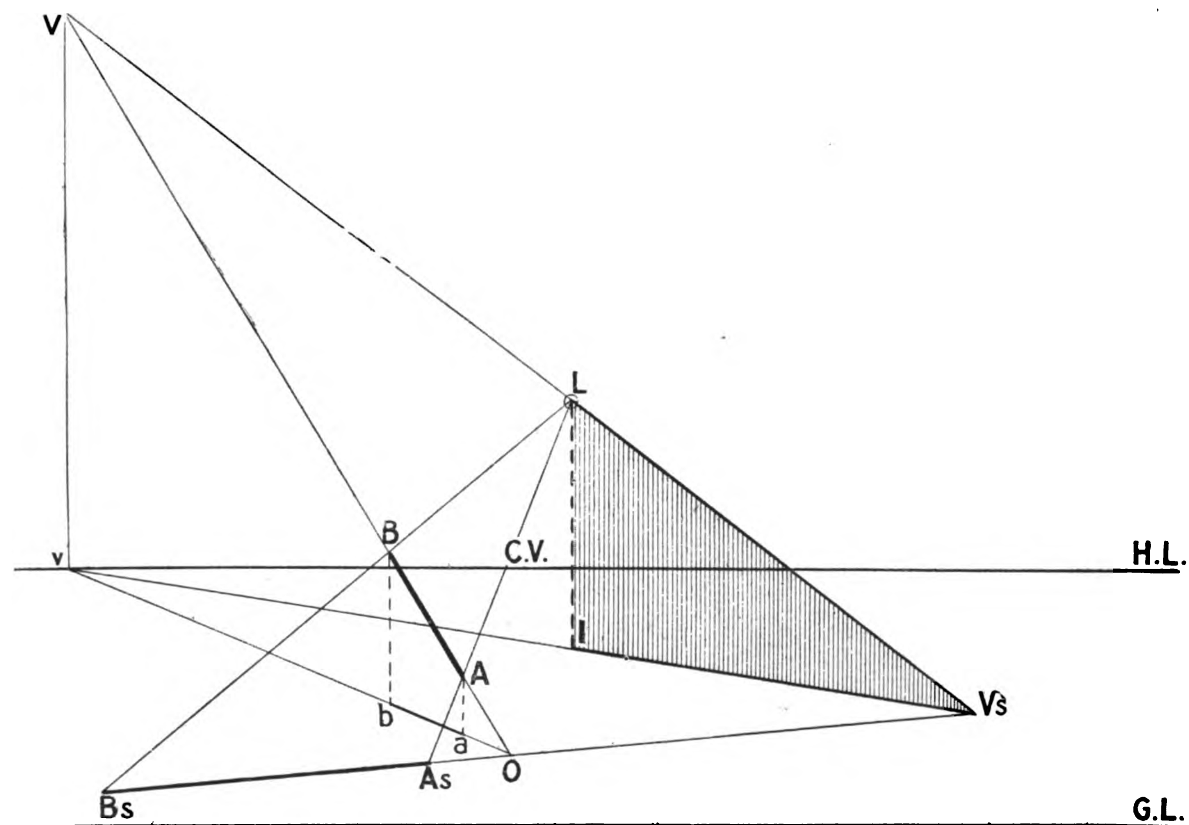
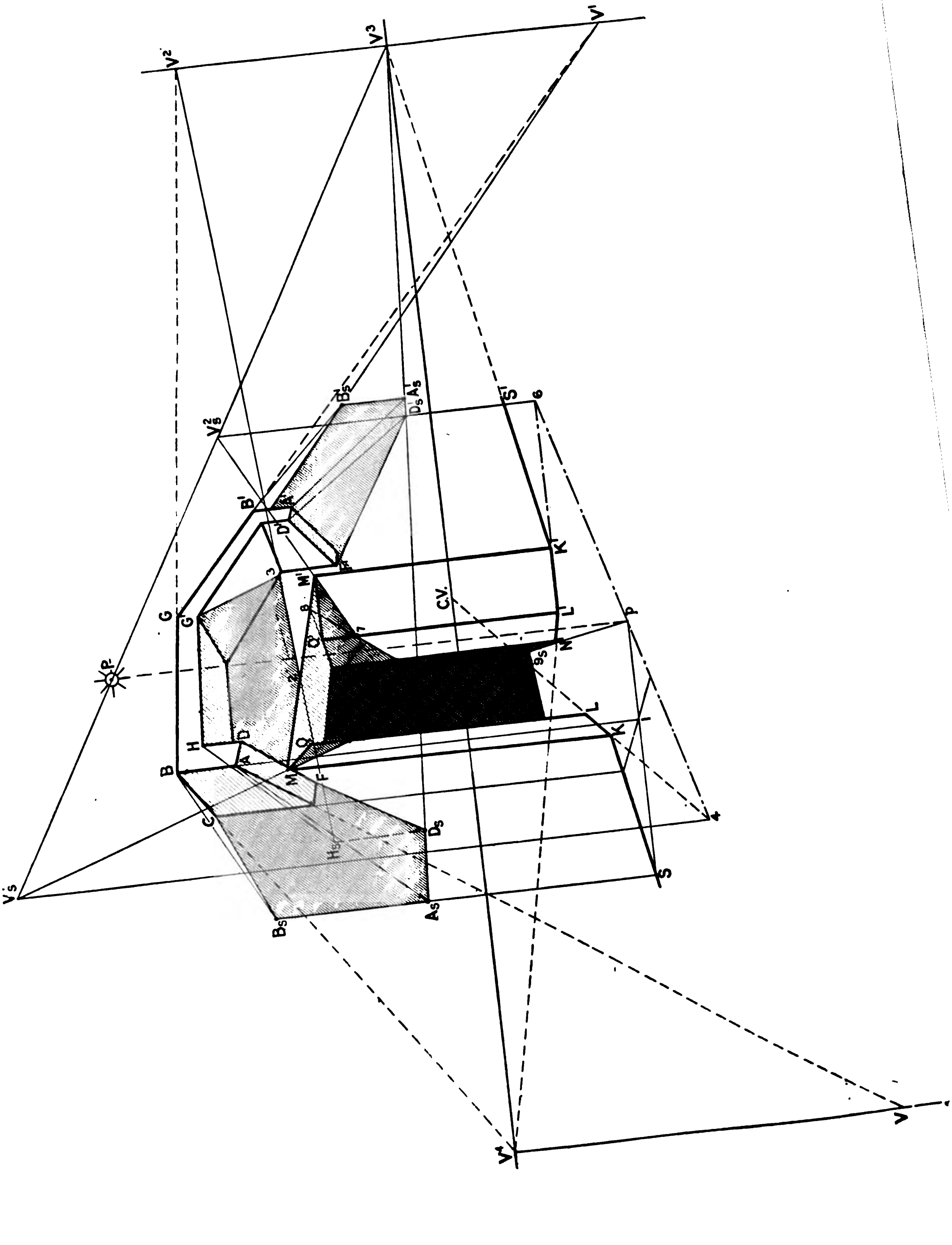


Fig. 32.

In fig. 32 **A B** is an oblique line vanishing at **V** and cutting the ground at **O**; **L** is an artificial light above **l** on the ground. Applying Rule 6, **V L** V_s is a line through **L** parallel to **B A**. The plane **L l** V_s cuts the ground along **v l** V_s, and hence **V L** cuts the ground at V_s. V_s is therefore the **V.P.** of **A B**'s shadow on the ground.

The figure also shows how the shadow may be determined; it passes through V_s and **O**.

PROBLEM XXVI.



PROBLEM XXVI.

(From an Examination Paper.)

The diagram gives the perspective representation of a doorway. Show the shadow cast by a luminous point **P** *vertically over* **p** *on the ground.* (**V^3** *and* **V^4** *are distance points.*)

1 is on the ground underneath **A** and **B**. The shadow of **1 A B** on the ground is **p 1 S**, on the vertical wall it is **S $A_S B_S$**. Determine **A_S** and **B_S** by drawing the rays **P A** and **P B**. Join **$B_S C$**.

The shadow of **A D** is on a plane parallel to itself and therefore it vanishes at **V^3**. As **A D $D^1 A^1$** is a straight line the shadow of each of these points lies on **$A_S D_S V^3$**. Determine them by drawing the rays.

Join **$D_S F$** and **$D^1_S F^1$**.

$B^1 G$ casts its shadow at **$V^1 B^1_S$** produced (Rule 5).

D H is vertical, hence **$D_S H_S$** is vertical, find the position of **H_S**. **H G^1** casts its shadow at **$H_S 2$**, **3 V^2** (Rule 5). **$H_S V^2$** determines **2** on **M M^1**, and **3**; join **3 G^1**.

Find the shadow of **M M^1**.

M M^1 casts a shadow on **K L Q M**. **V^1_S** is the **V.P.** of that shadow. (**P V^1_S** is a line through **P** parallel to **M M^1**, **p 4** is the plan of this line, **4** being the plan of the intersection of **P V^1_S** with **K L Q M** produced.)

Produce **$V^1_S M$** to cut **Q L** at **5**.

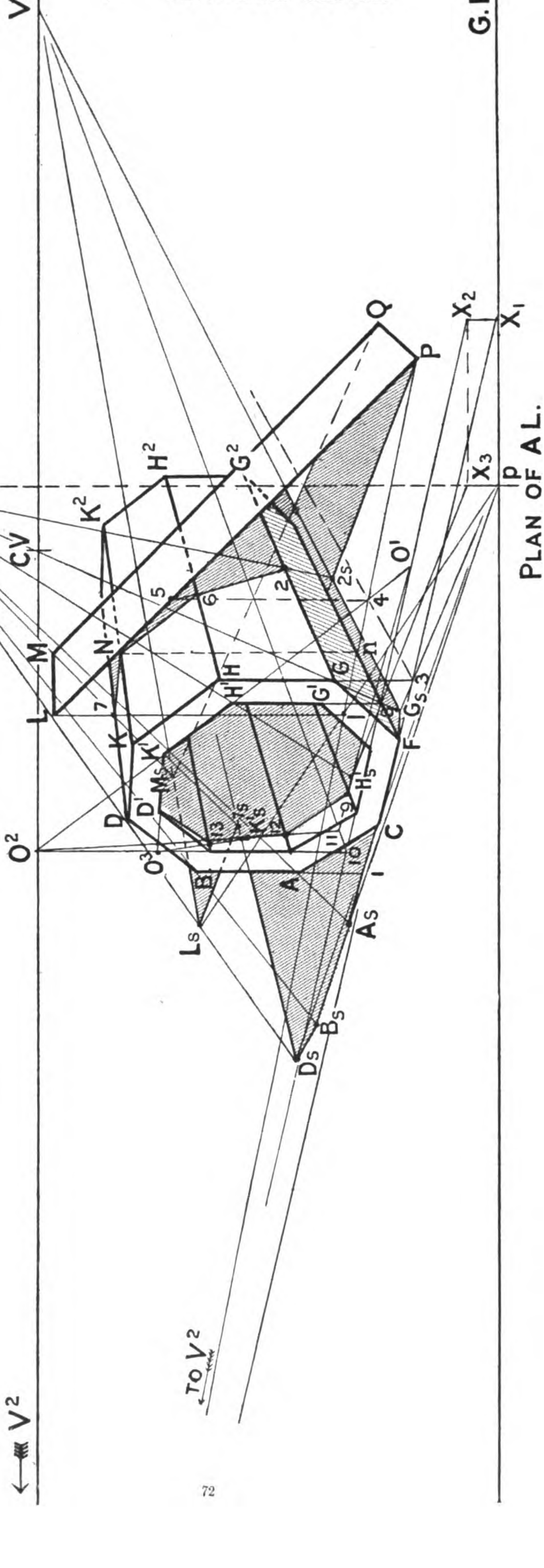
Similarly **V^2_S** is the **V.P.** of the shadow cast by **M M^1** on **$K^1 L^1 Q^1 M^1$**. **$V^2_S M^1$** produced determines **7**.

(Notice that **5,7** should vanish at **V^3**.)

8 is the intersection of **M M^1** with **$L^1 N^1 R^1 Q^1$**. **8,7,9** is the shadow of **M M^1** on that plane.

$N^1 R^1_S$, the shadow of **$N^1 R^1$** runs in the direction **p $N^1 R^1_S$**.

The shadow cast within the doorway is determined by drawing the ray **P 9** to cut **$N^1 R^1_S$** at **9_S** and producing **$V^3 9_S$** till it cuts **L Q**.


V'
G.L.
A L
C.V
PLAN OF A L.
O²
V²
TO V²

PROBLEM XXVII.

(From a Recent Examination Paper.)

The diagram gives the perspective representation of a hollow octagonal prism, with a board resting against it. Show all the shadows cast by an artificial light in the position shown in the diagram.

(*a*) Find the shadow cast by the prism on the ground.

Determine V^1 and V^2 the V.P.s of the long edges of the prism and C F respectively. 1 is on the ground underneath A and B.

p (plan of A L) 1 produced gives the position of the shadow of B A 1; rays from A L, through A and B, determine A_S and B_S.

As C is on the ground A_SC is the shadow of A C.

The vertical line C D casts p C produced; draw the ray through D to cut p C at D_S: join B_SD_S.

The long edge from D now casts its shadow at D_SV^1.

3 is on the ground underneath G.

p 3 produced is the position of the shadow of 3 G on the ground; the ray through G determines G_S on this line; join G_SF (the shadow of G F on the ground).

The long edge through G casts its shadow at G_SV^1.

(*b*) Find the shadow cast by the board.

P Q vanishes at V^1 as the board rests against an edge of the prism; determine if the board rests against $H H^2$ or $K K^2$.

P n vanishing at V^2 is the plan of P N L. P n cuts $3 V^1$ (the H.T. of $H H^2G^2G$) at 4. 4,5, a vertical line through 4 gives 5 on P L as the intersection of P N L with the vertical surface $H H^2G^2G$ (produced). As 5 does not come on $H H^2$, the board does *not* rest against $H H^2$. P n cuts $F V^1$ (the H.T. of a vertical plane containing $K K^2$) at n. n N, a vertical line through n gives N on P L as the intersection of P N L with the vertical plane containing $K K^2$. N also lies on the edge $K K^2$, and therefore the board touches the prism at N (the line of contact will be along $K K^2$).

From L draw a projector to the ground, cutting it on P n l at l. Find L_S, the shadow of L on the ground; join L_SP.

L_SP is the shadow of L P on the ground.

L_SM_S vanishes at V^1 (as L M vanishes at V^1), the ray through M determines M_S; join M_SQ, M_SQ is the shadow of M Q on the ground.

P L_S cuts G_SV^1 at 2_S. 2_S is cast by 2 on $G G^2$. 2,5 is the shadow of P L on $H H^2G^2G$ (Rule 4) until it meets the edge $H H^2$ at 6. 6 N is the shadow of P L on $H H^2K^2K$ (Rule 4). L_SP cuts D_SV^1 at 7_S which is cast by 7 on $D V^1$. 7 N is the shadow cast by L N on the top of the prism.

(*c*) The shadow cast *within* the hollow prism.

Produce V^2C 3 to cut the P.P. at X_1, and 10,8 to cut the P.P. at X_2. Make p X_3 equal to X_1X_2. X_3 is the intersection with A L p of a horizontal plane containing 10,8.

H^1G^18 casts X_38, H^1_S is obtained on X_38 by drawing the ray. K^1H^1 cuts a horizontal plane containing 10,8 at O^1; and a vertical plane containing the internal surface which is next and parallel to A B at O^2; D^1K^1 cuts the same vertical surface at O^3.

The shadow of H^1K^1 runs from O^1 through H^1_S and 9 to 11, (9,11 is used for construction). 11 O^2 is its shadow, 11 O^2 gives 12, K^1_S being found on 11 O^2 by drawing the ray. Join 9,12. $K^1_SO^3$ determines 13 (Rule 4). Join 13 D^1 (Rule 4).

This completes the visible portion of the shadows cast by the object.

REFLECTIONS.

The reflections usually required are those given by *plane* mirrors in horizontal or vertical positions, so that these only will be considered.

The following laws are applicable to reflections in plane mirrors.

1. A point and its reflection lie on the same perpendicular to a reflecting surface, equidistant from, but on opposite sides of that surface.

2. The reflection of a line runs towards the intersection of the line with the reflecting surface.

3. A line and its reflection lie in the same plane, and the angle between the reflection and the reflecting surface is equal to the angle between the line and that surface.

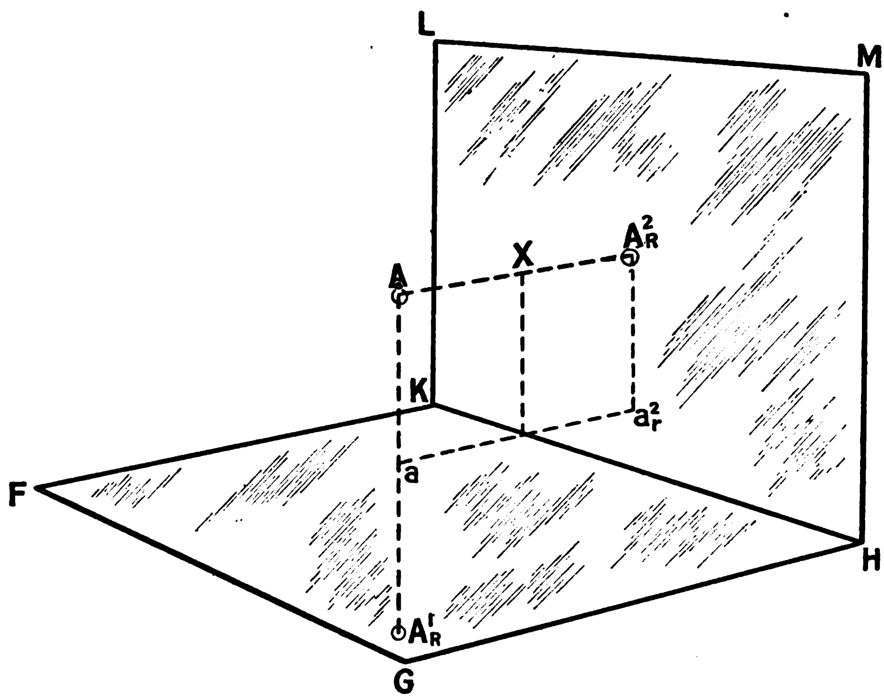


Fig. 33.

In fig. 33 **F G H K** is a horizontal mirror, and **A** is a point vertically above **a**, which is on the mirror. The image of **A** in **F G H K** would appear at A_R^1. AA_R^1 being perpendicular to **F G H K**, and $A_R^1 a = Aa$.

The image of **A** in the vertical mirror **H K L M** would appear at A_R^2. AA_R^2 being perpendicular to **H K L M**, cutting it at **X**, and XA_R^2 being $= XA$.

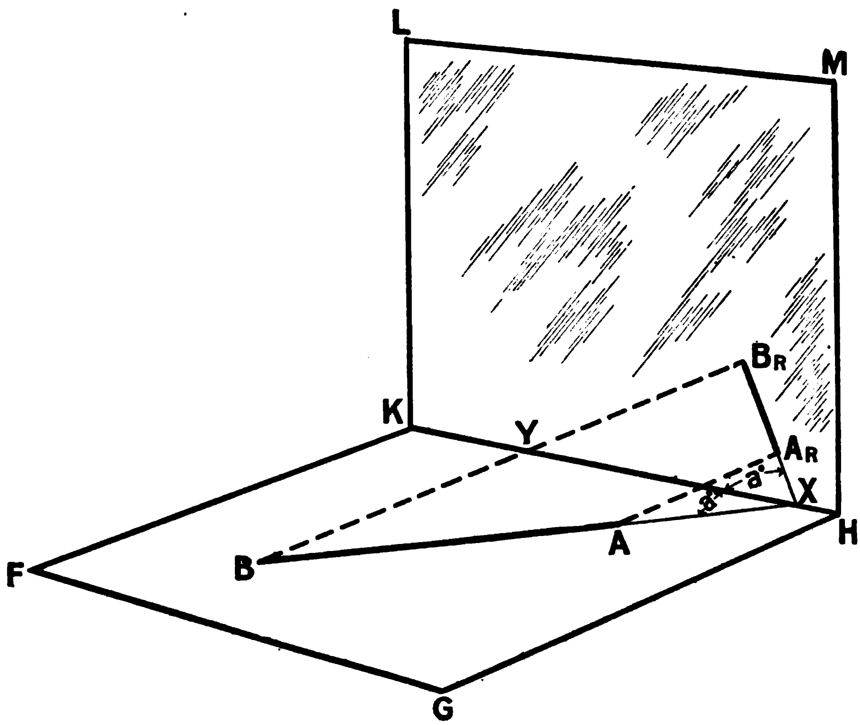
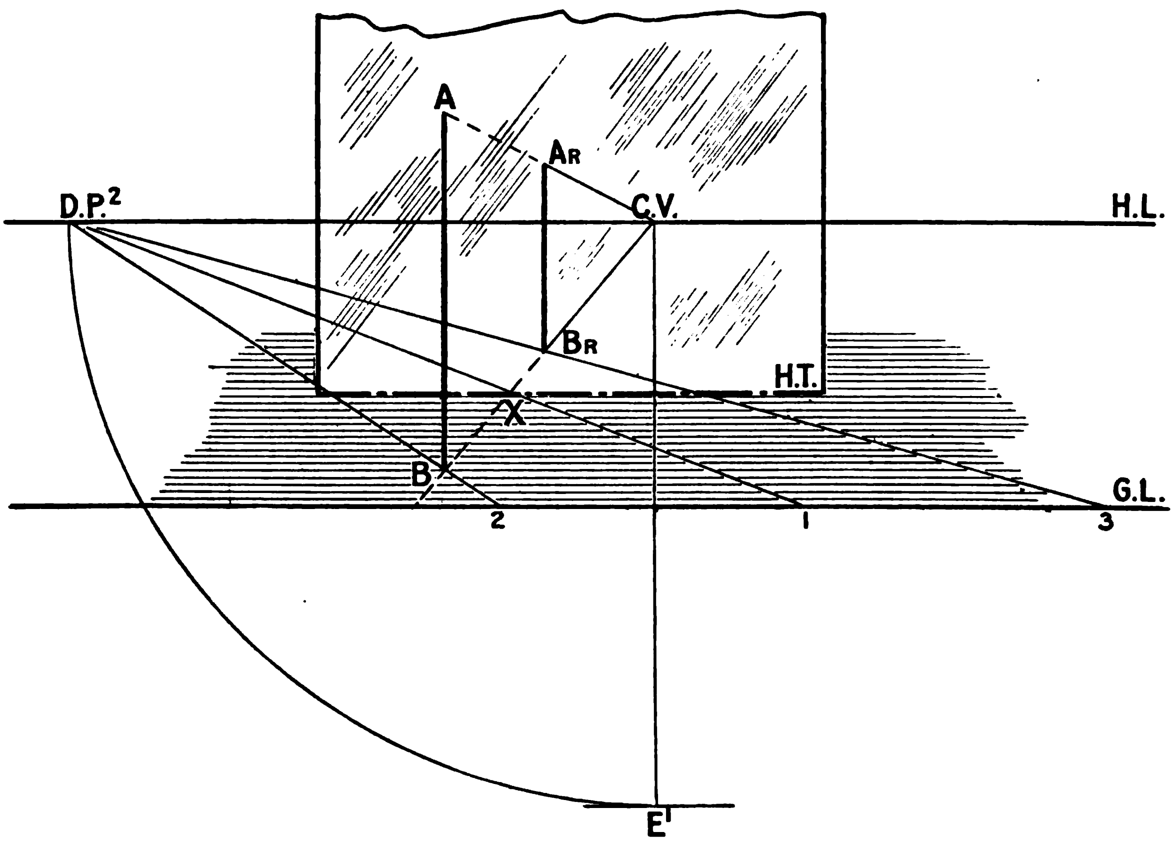


Fig. 34.

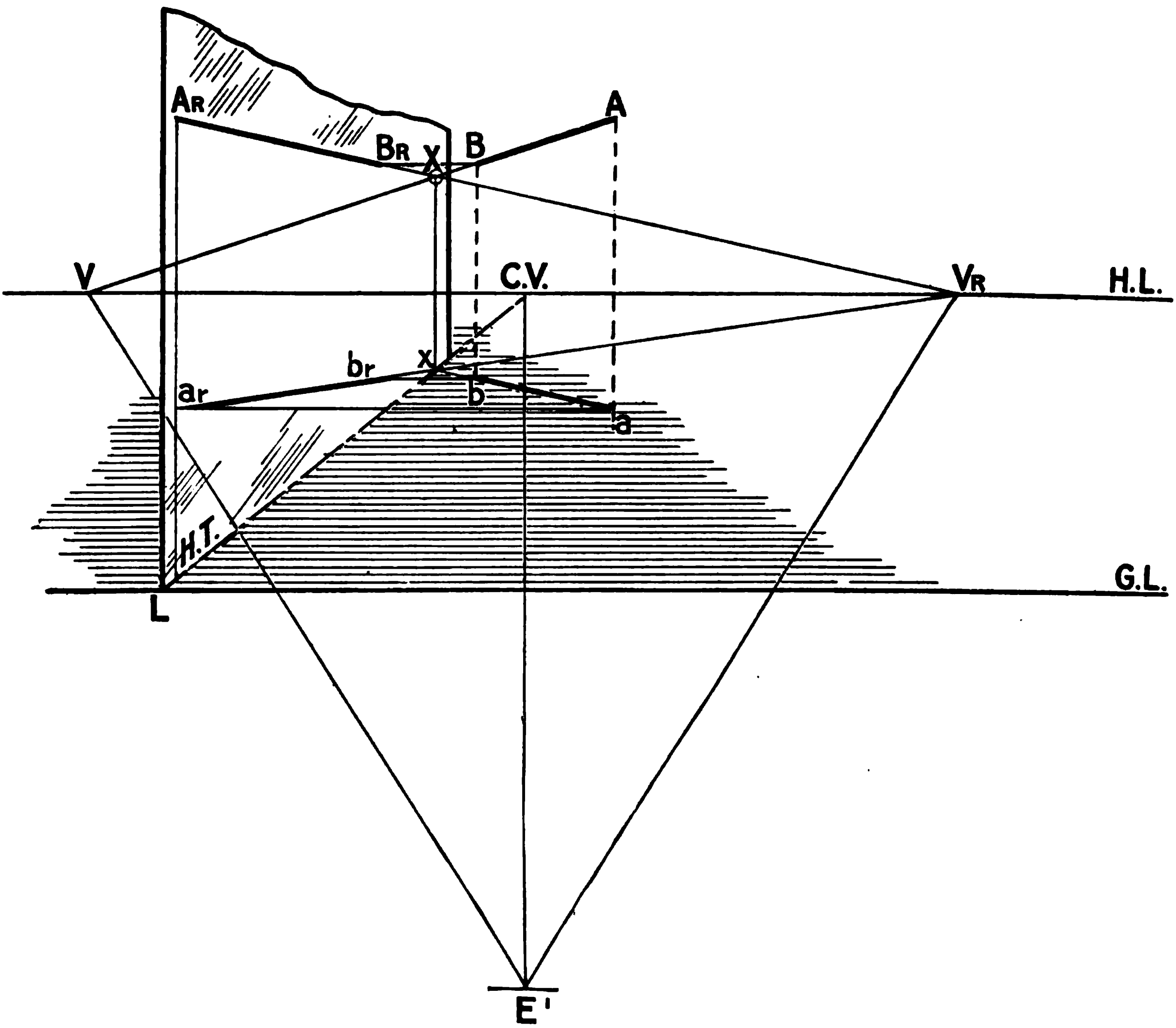
In fig. 34 **A B** is a line lying on the ground (**F G H K**). **H K L M** is a vertical mirror. **A B** cuts **H K L M** at **X**, the reflection of **A B**, *i.e.* $A_R B_R$ on being produced would also cut **H K L M** at **X**. The $\angle$ $B_R X K$ is = $\angle$ **B X K**. $B B_R$ and $A A_R$ are perpendicular to **H K L M**, $B_R Y$ being = **B Y**.



PROBLEM XXVIII.

AB *is a vertical line,* **H.T.** *the horizontal trace of a vertical mirror parallel to the* **P.P.** *Find the reflection of* **AB** *in the mirror.*

A A_R and **B** B_R will vanish at **C.V.** (Rule 1). B_R**X** is equal to **BX,** do this by using **D.P**^{2}. as a measuring point (Rule 1).



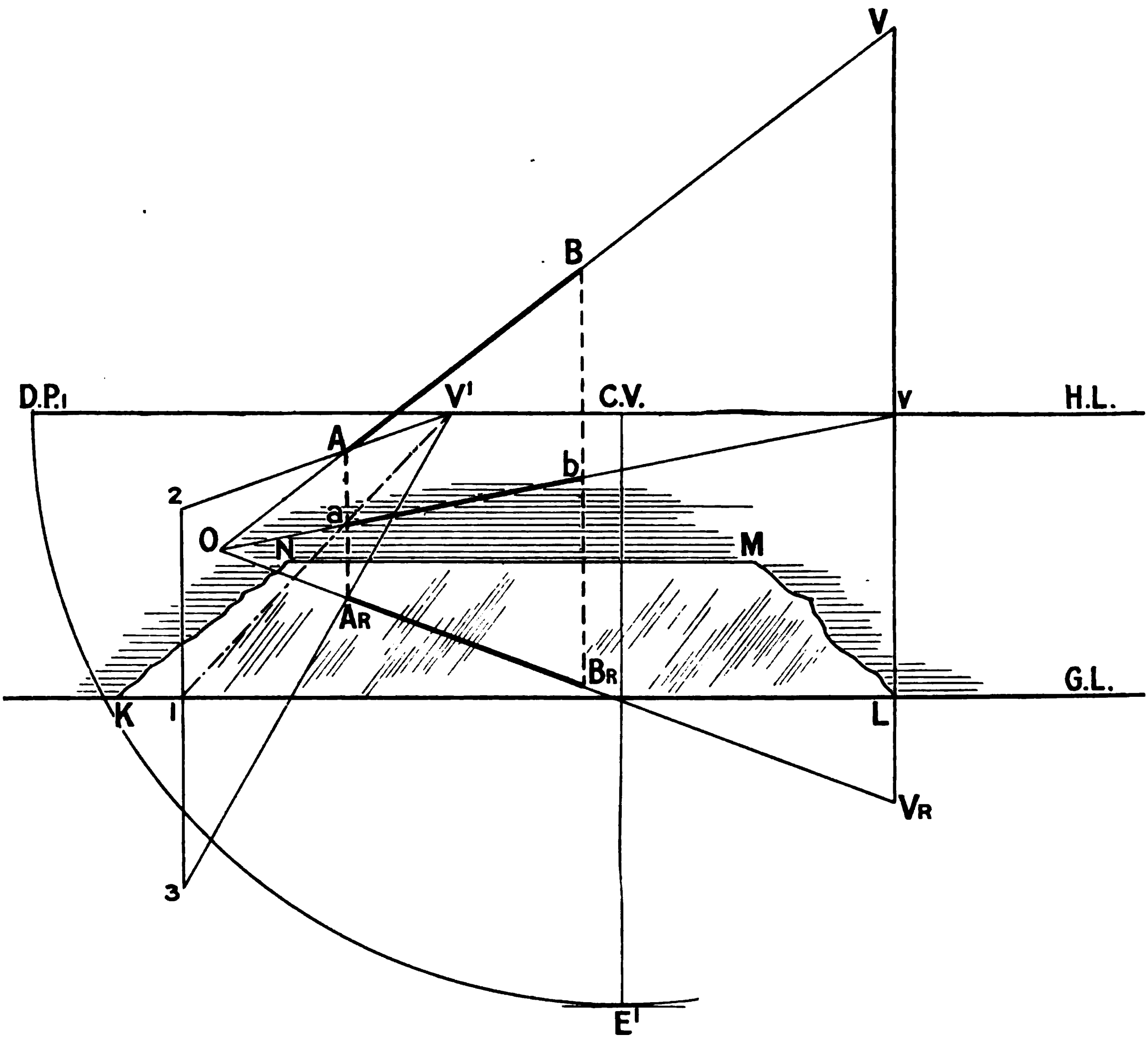
PROBLEM XXIX.

A B *is a horizontal line vanishing at* **V.** **H.T.** *is the horizontal trace of a vertical mirror perpendicular to the* **P.P.** *Find the reflection of* **A B** *in the mirror.*

E¹ C.V. is the vanishing parallel of the **H.T.** of the mirror. $\angle$ **a_rx L** must be equal to **a x L** (Rule 3), thus **V E¹ C.V.** will be equal to **V_RE¹ C.V.** **E¹V** and **E¹V_R** being the vanishing parallels of **a b** and **$a_r b_r$** respectively. It will be evident in this case that **C.V. V_R** will be equal to **C.V. V.** **V_R** being the reflection of the **V.P.** of **a b** and **A B.**

Determine **X** the intersection of **A B** with the mirror (it is vertically above **x**). Produce **V_RX.**

A_R and **B_R** lie in perpendiculars to the mirror from **A** and **B** respectively, these perpendiculars will be parallel to the **P.P.** in this problem.



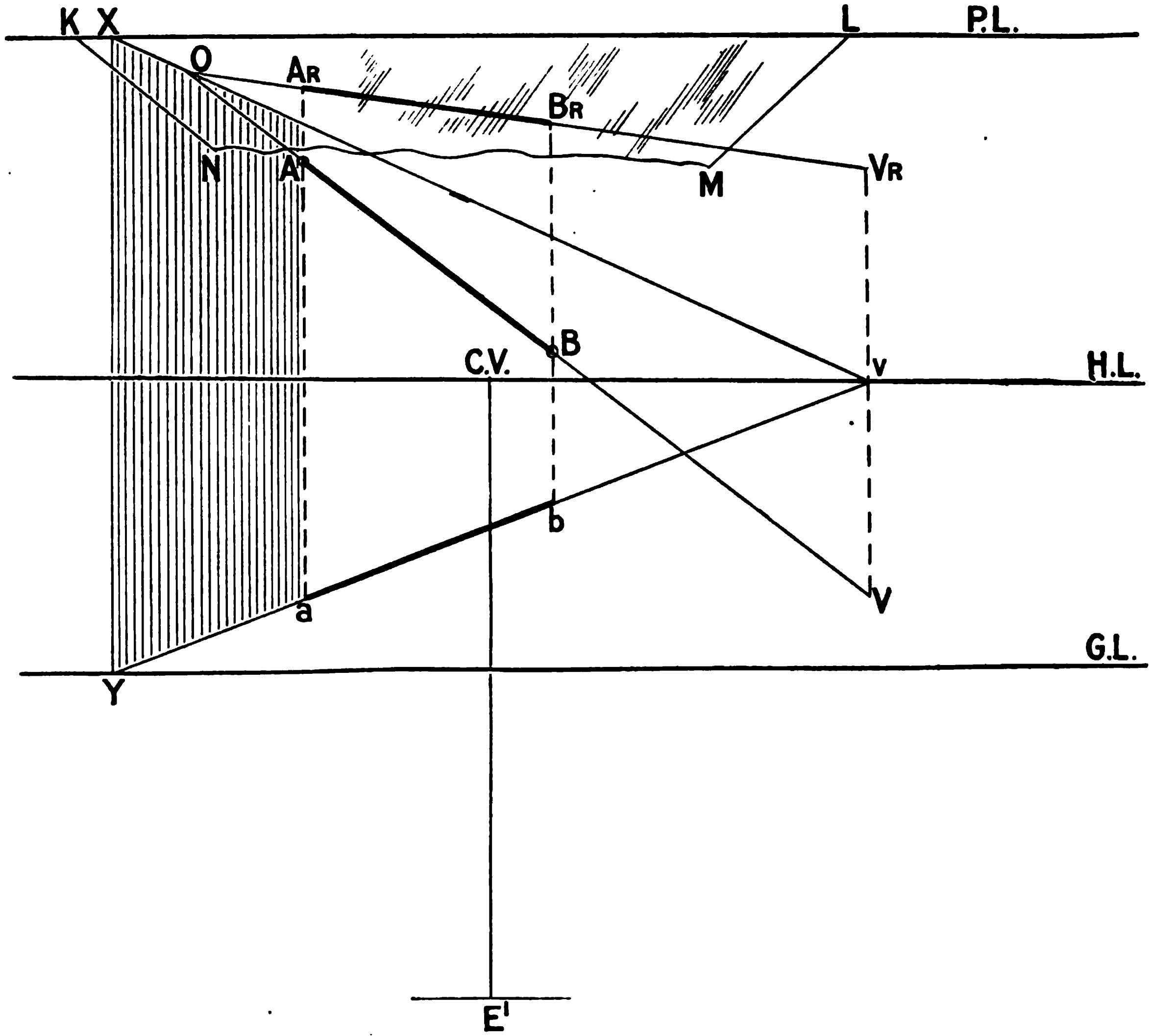
PROBLEM XXX.

A B *is an oblique line intersecting the ground at* **O** *and vanishing at* **V**. **K L M N** *is a horizontal mirror lying on the ground.*

Find the reflection of **A B** *in* **K L M N**.

As **A B** cuts the ground at **O** it therefore also cuts the plane of the mirror at **O**; hence the reflection of **A B** in **K L M N** passes through **O** (Rule 2).

The $\angle B_R O b$ will be equal to the $\angle BOb$ (Rule 3), and, as **a b** is on the ground, $V_R v$ will be $= V v$. Join $O V_R$; from **A** and **B** draw perpendiculars to the mirror and obtain A_R and B_R. (In this problem the perpendiculars are vertical lines.) It is useful to note that if a vertical plane is supposed to contain **A a**, $V^1 a 1$ is its **H.T.**, **3,1,2** its **P.L.**, that the horizontal line $V^1 A 2$ cuts **P.P.** at **2**. The reflection of **2** is at **3**, **1,3** being equal to **1,2**. $3 V^1$ is the reflection of $2 V^1$ and passes through A_R. Also that $A_R a$ will be equal to **A a** when drawn in perspective.



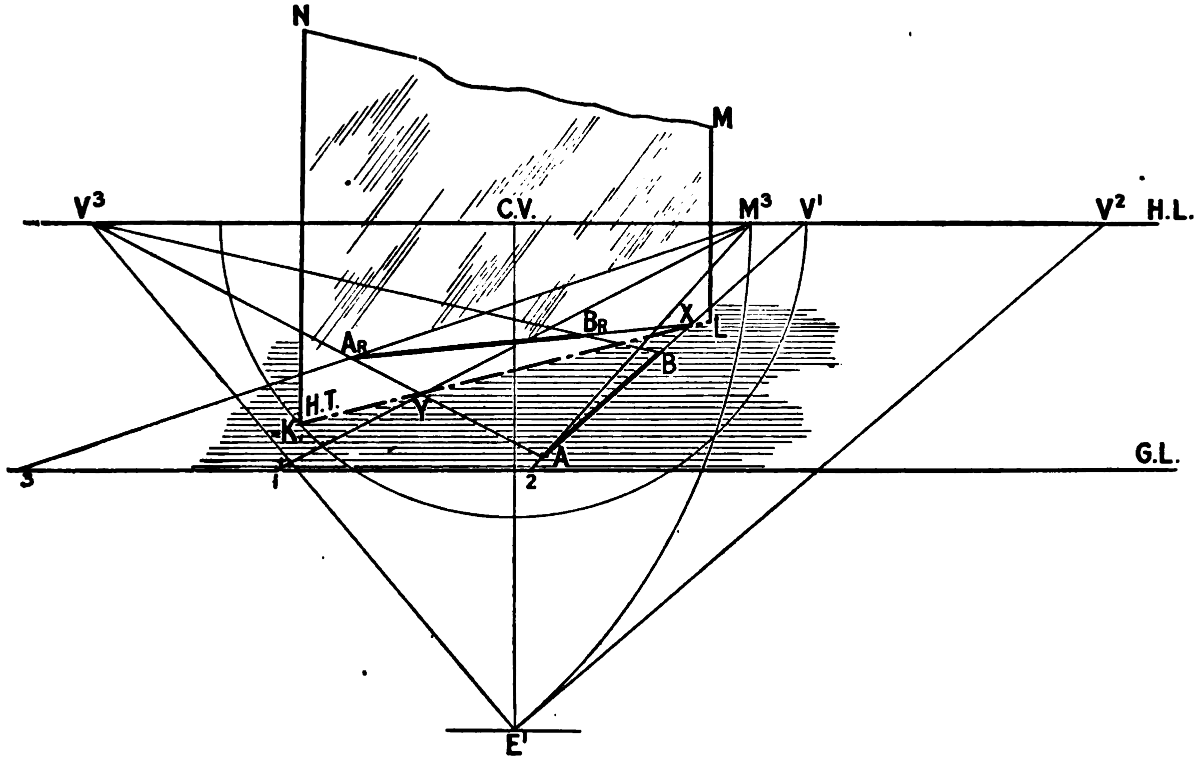
PROBLEM XXXI.

A B *is an oblique line vanishing at* **V.** **a b** *is the plan of* **A B.** **K L M N** *is a horizontal mirror* above *the eye. Find the reflection of* **A B** *in* **K L M N.**

Obtain **O,** the intersection of **A B** with the mirror. (**A a Y X** is a vertical plane containing **A B**; this plane intersects the ground at **v a Y** and the mirror at **X v.** **A B** cuts the mirror at **O** in **X v.**)

Make $V_R v$ equal to **V v** (observe that V_R comes above the **H.L.**). Join $O V_R$.

$A A_R$ and $B B_R$ the perpendiculars to the mirror must be vertical lines as the mirror is horizontal.



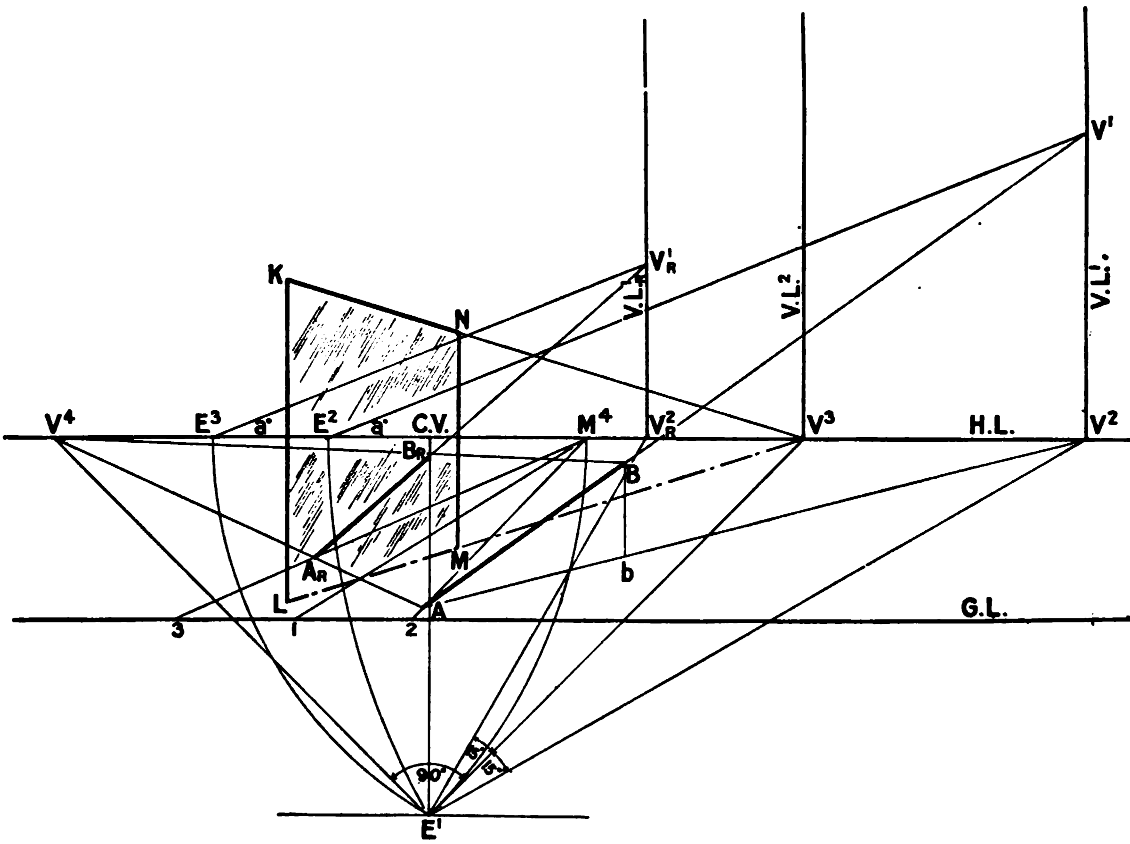
PROBLEM XXXII.

A B *is a line lying on the ground and vanishing at* **V^1**. **K L M N** *is a vertical mirror; its horizontal trace vanishes at* **V^2**. *Show the reflection of* **A B** *in* **K L M N**.

Find V^3 the V.P. of lines perpendicular to the mirror by making $V^2E^1V^3 = 90°$; M^3 is the M.P. of V^3.

Join AV^3 and BV^3; the reflections of A and B lie in these lines (Rule 1). Produce M^3Y and M^3A to the G.L. at 1 and 2 respectively. Measure off 1,3 equal to 1,2, join $3\,M^3$, cutting AV^3 at A_R (Rule 1).

A B cuts the mirror at X; join XA_R, cutting BV^3 at B_R.



PROBLEM XXXIII.

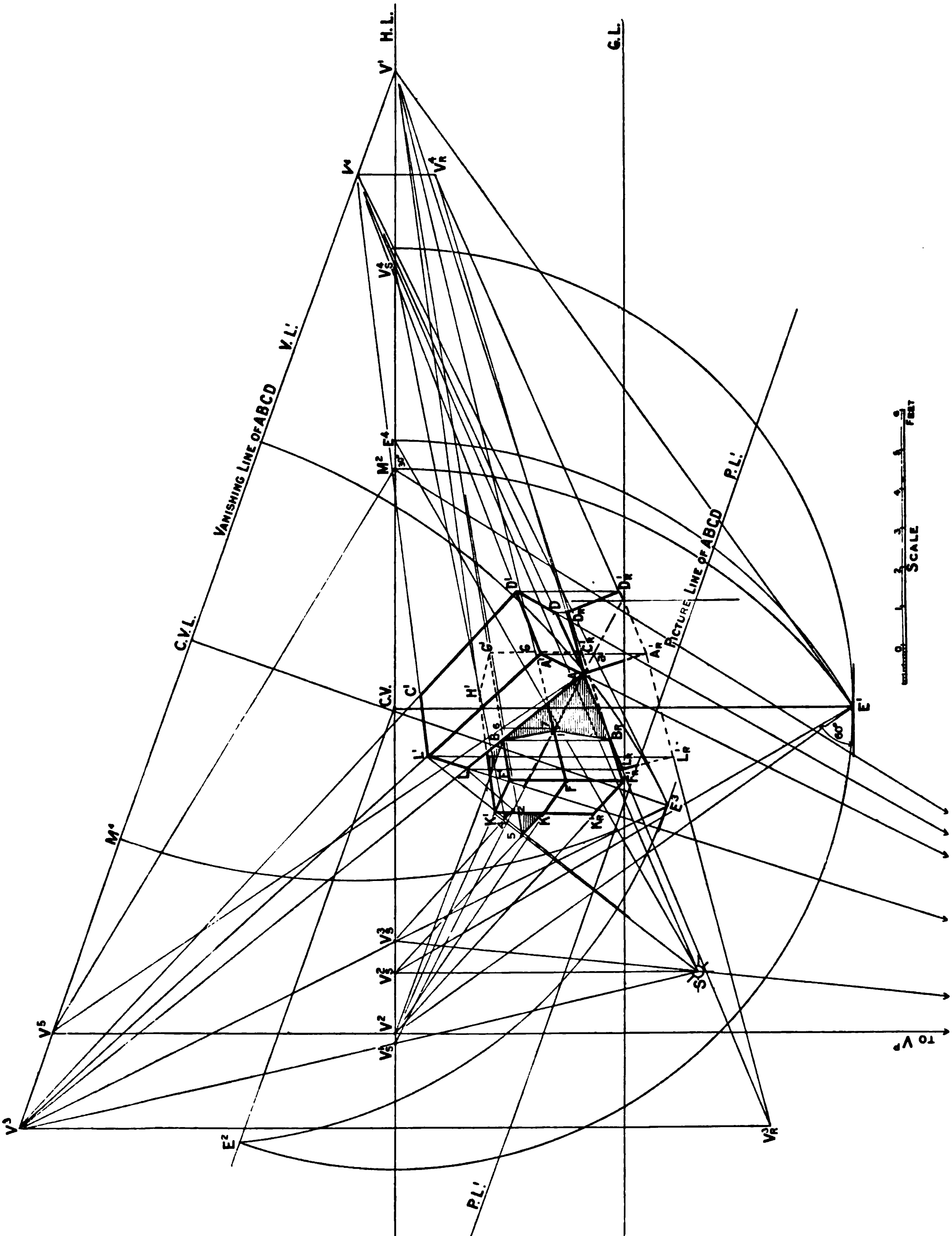
A B *is an oblique line, with one end* **A** *on the ground.* **A B** *vanishes at* V^1. **K L M N** *is a vertical mirror, inclined to the* **P.P.** *Find the reflection of* **A B** *in* **K L M N.**

The reflection of **A** is obtained as in Problem XXXII.

To find the **V.P.** *of* **A B***'s reflection*:—E^1V^3 is the vanishing parallel of the mirror's **H.T.**

The line beneath **A B** on the ground would vanish at V^2, the reflection of this line would vanish at V^2_R (the $\angle V^2E^1V^3$ being equal to $\angle V^3E^1V^2_R$ as in Problem XXXII). The reflection of $V.L.^1$ is therefore a vertical line through V^2_R, *i.e.* $V.L.^1_R$. The **V.P.** of **A B**'s reflection must be in $V.L.^1_R$. Rotate the eye about V^1V^2 into the **P.P.** at E^2, then $V^2E^2V^1$ is **A B**'s inclination. As the inclination of **A B** remains the same in the plane containing its reflection, rotate the eye about $V.L.^1_R$ into **P.P.** at E^3. Set off **A B**'s inclination, giving V^1_R on $V.L.^1_R$.

The reflection of **A B** vanishes at V^1_R, and it is terminated by the perpendicular to the mirror from B, *i.e.* BV^4.


H. L.
G. L.
VANISHING LINE OF ABCD
V. L'.
C.V.L.
C.V.
PICTURE LINE OF ABCD
P.L'.
SCALE
FEET
TO V^P

GENERAL PROBLEMS.

PROBLEM XXXIV.

(From a Recent Examination Paper.)

(*a*) *Represent in perspective a rectangular solid* 2 *ft. high,* 3 *ft. wide, and* 6 *ft. long, resting on one of its largest faces on the horizontal plane, its length vanishing towards your right at an angle of* 35° *to the picture plane, and its nearest angle on the ground being* 6 *ft. below the eye,* $2\frac{1}{2}$ *ft. to the left of the centre, and* 4 *feet within the picture. Afterwards find two points, the first,* **A**, *on the ground* 1 *ft. to the right of the centre and* $2\frac{1}{2}$ *ft. within the picture, and the second,* **B**, *on the near top edge of the solid at a distance of* $1\frac{1}{2}$ *ft. from the nearest corner, draw a line from point* **A** *through point* **B**, *measure it* 6 *ft. long, and make this line one edge of a solid similar in every respect to the first solid, and resting upon it and with one angle on the ground. Distance to the spectator* 12 *ft. Scale* $\frac{1}{2}$ *in. to* 1 *ft.*

(*b*) *Show the shadows cast by the solids when the light streams over the right shoulder of the spectator in vertical planes at* 60° *to the picture plane and is inclined at* 30° *to the ground plane.*

(*c*) *Show the reflections in the horizontal planes.*

(*a*) **K F K^{1}F^1 G G^{1}H^1** is the perspective representation of the first block.

A B is the required line forming an edge of the second block. It must be carefully noted that **A B** is not necessarily at right angles to the **H.T.** of the plane containing the lower face of the second block (in this case it is *not* at right angles), and that the **H.T.** must be parallel to **F G** as the block rests on **F^{1}G^1**.

Find the **V.L.** of the plane containing the lower face of the second block.

V^1 is this plane's **V.P.** of direction.

E^{1}V^2 is the vanishing parallel of the **H.T.** of a vertical plane cutting the plane of the lower face of the block at right angles to its **H.T.**

This vertical plane, of which **V^{2}V^5** is the **V.L.**, cuts the ground at **A V^2**, the side of the block in **7,6**, and the oblique plane containing the lower face of the upper block in **A 6** produced.

A 6 vanishes at **V^5** on **V^{2}V^5**. **V^{5}V^1** is the required vanishing line.

A B cuts **V^{1}V^5** produced at **V^3**, the **V.P.** of **A B**.

Determine **E^3** the position of the eye when rotated into the **P.P.** about **V^{1}V^5**. **V^{3}E^{3}V^4** is a right angle and gives **V^4** as the **V.P.** of **A D**.

V^{5}M^{2}V^P is a right angle, and this determines the position of **V^P**. The perspective representation of the blocks should present no further difficulty.

(*b*) Observe that the sun is *behind the spectator;* **S** is the **V.P.** of its rays.

V^2_S and **V^1** are the **V.P.**s of the shadows cast by **K K^1** and **K^{1}H^1** respectively.

The shadow of **A B L** on the ground vanishes at V^1_S; this, on drawing rays, gives **1** on **F G** and **2** behind the first block. **2,4** vanishes at V^2_S; **4** is joined to **V^1**.

Join **1 B** (as **B** is on **F^{1}G^1**) and **B** V^1_S.

This completes the visible shadows cast by the blocks.

(*c*) The reflections of **F**, **K**, and **A** coincide with **F**, **K**, and **A** respectively. **F** F^1_R is equal to **F F^1**. $F^1_R K^1_R$ vanishes at **V^2**.

The reflection of **F^{1}G^1** vanishes at **V^1**.

Join **A** V^4_R and **A** V^2_R; from **D** and **L** draw vertical lines to cut **A** V^4_R and **A** V^2_R at D_R and L_R respectively.

a^1 is on the ground below **A^1**.

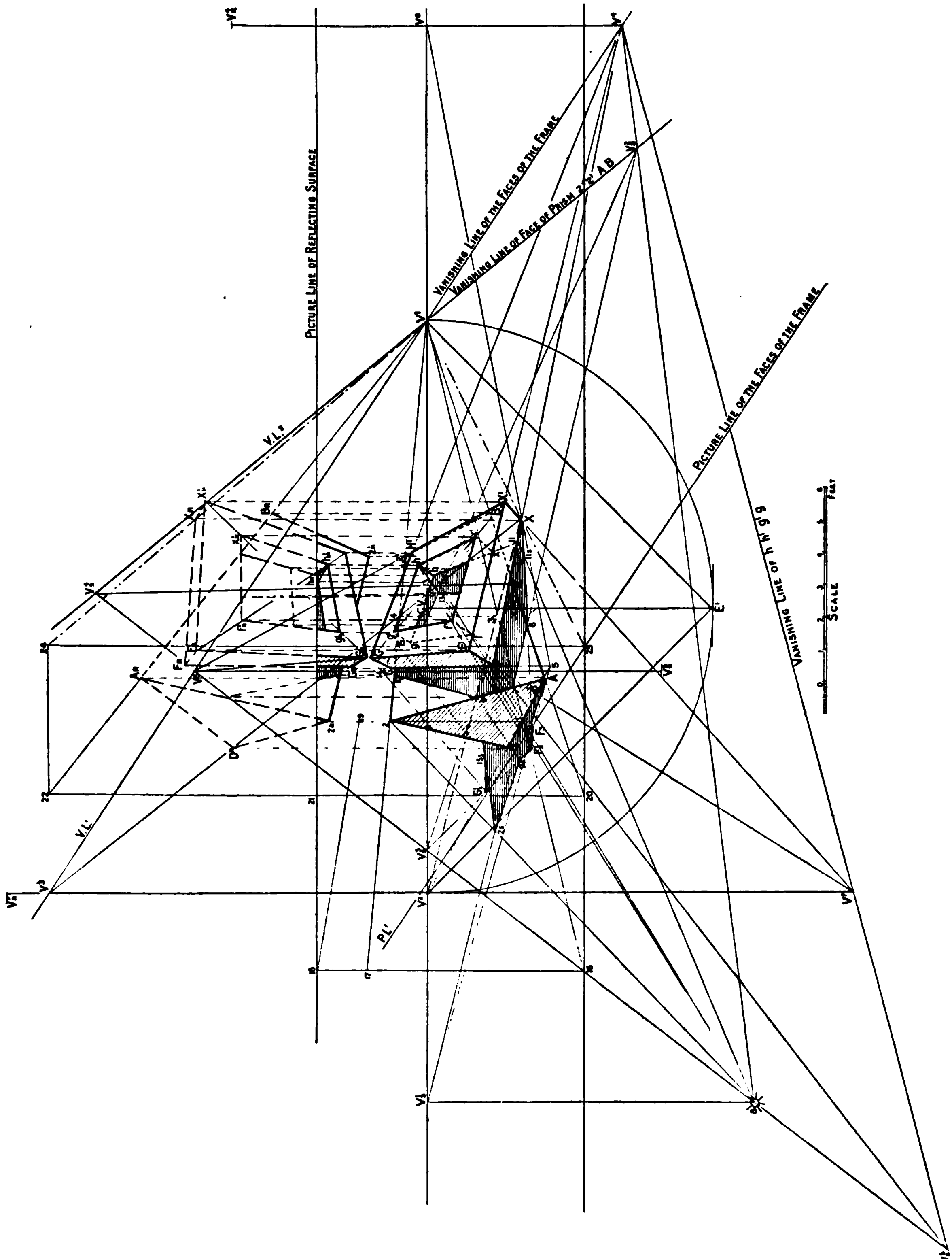
a^1A^1_R is equal to **A^1**a^1.

$A^1_R D^1_R$ and $A^1_R L^1_R$ vanish at V^4_R and V^2_R respectively, perpendiculars to the mirror determine D^1_R and L^1_R.

Join $L^1_R L_R$ and $D^1_R D_R$.

B's reflection is in the reflection of **F^{1}G^1** and in a vertical line through **B**, *i.e.* at B_R.

Finally B_R**1** is the reflection of **B 1**.


PICTURE LINE OF REFLECTING SURFACE
VANISHING LINE OF THE FACES OF THE FRAME
VANISHING LINE OF FACE OF PRISM 2 2' A B
PICTURE LINE OF THE FACES OF THE FRAME
VANISHING LINE OF h h' g' g
SCALE
FEET
V.L.2
V.L.1
P.L.1

PROBLEM XXXV.

(FROM A RECENT EXAMINATION PAPER.)

(*a*) *Represent in perspective a triangular prism, 8 ft. wide by 15 ft. long, on a horizontal plane 5 ft. below the eye, its nearest angle being 3 ft. to the left of the centre and 3 ft. within the picture, with its long edges vanishing at angles of 45° to the ground line towards your right. Draw a line on the ground at a distance of 3 ft. from the nearest long edge and parallel to it, and upon this line mark a point* **X** *6 ft. from the ground line. With one angle upon point* **X** *and a face leaning on the top edge of the prism, put into perspective a square frame of 8 ft., with sides 1½ ft. square, one long edge sloping up from* **X** *towards the spectator at an angle of 25° to the horizontal trace of the plane of a face of the frame. Distance of the spectator, 9 ft. Scale ½ in. to 1 ft.*

(*b*) *Show the shadows cast by these solids when the rays of light vanish downwards towards your left at 30° to the ground in planes at 30° to the picture plane.*

(*c*) *Show the reflection of the two solids in a horizontal plane above them 8½ ft. from the ground.*

(*a*) Draw the triangular prism in the given position.

23 X is a line 3 ft. from **A B**, and **X** is 6 ft. from the **G.L.** (find any point 6 ft. from the **P.P.**, by drawing a horizontal line through this point, all points 6 ft. beyond the **P.P.** are obtained; this line cuts **23 X** at **X**, the required point). As the frame rests with part of its surface against **2,2**^{1}, the **H.T.** of the plane containing the frame's lower face vanishes at V^1; and it passes through **X** as **X** is on the ground.

V.L.^{1} is the vanishing line of the plane containing the frame's face; **V.L.**^{1} is obtained in the following manner:—

X V^2 is the **H.T.** of a vertical plane cutting the lower face perpendicular to its **H.T.** **X** V^2 cuts **1** V^1 at **3** (**1** V^1 is on the ground underneath **2,2**^{1}). **3,4**, a vertical line, gives **4** in **2,2**^{1}. **X 4** is the intersection of the assumed plane with the frame's face; as this line lies on the assumed plane it vanishes at V^3 above V^2. Join V^3V^1.

P.L.^{1} is the picture line of the plane containing the frame's lower face.

V.L.^{2} is the vanishing line of the side of the prism.

In obtaining f^1, x^1, h^1, and g^1, draw the diagonals G^1X^1 and F^1H^1.

(*b*) **S** is the **V.P.** of the sun's rays.

$1,2_S$ is the shadow of **1,2**; join 2_SA and 2_SV^1, this determines the shadow of **2 A** and **2,2**^{1}.

V^6X produced gives a line on the ground below **X F** (V^4V^6 must be vertical).

F 5 drawn perpendicular to the ground gives **5** as the point on the ground below **F**.

$5\,F_S$ is the shadow **5 F**, giving F_S.

$F\,F^1$ cuts the ground at **8** on $5\,V^2$ (a vertical plane containing $F\,F^1$ cuts the ground at $5\,V^2$).

$8\,F_S$ produced determines $F_SF^1_S$.

$F^1_SG^1_S$ vanishes at V^3_S.

$X\,F_S$ cuts **A B** at **6**, the shadow of F^1G^1 then runs on the prism, vanishing at V^2_S.

9_S is cast by **9**. **9,10,14**, the shadow of F^1G^1, vanishes at V^1_S. (**14** is on the prism produced.)

14,15, the shadow of G^1H^1, vanishes at V^2_S, giving **15** on **2,2**^{1}. **15** casts its shadow at 15_S on 2_SV^1. Join $G^1_S15_S$.

h x cuts **X F** at **11**. **11** casts its shadow at 11_S on **X 6**; the shadow of **11 h** is 11_S12, which vanishes at V^3_S, then **12** h_S on the prism.

The shadow of h^1g^1 vanishes at V^2_S and passes through h_S. h^1x^1 casts a shadow on $g\,g^1h^1h$.

$g\,g^1$ and $h\,h^1$ vanish at V^P; g^1h^1 and **g h** vanish at V^4, hence V^4V^P is the **V.L.** of the plane $g\,g^1h^1h$.

V^5_S is obtained by producing V^5S to intersect V^4V^P.

h^113 vanishing at V^5_S is the shadow of h^1x^1 on $g\,g^1h^1h$. **13** casts its shadow at 13_S.

h^1x^1 casts part of its shadow on the prism from 13_S, *i.e.* $V^1_S13_S$ produced.

(*c*) Notice that perpendiculars to the mirror are vertical lines. **18,21** is the **P.L.** of the reflecting surface.

A vertical plane containing **2,2**^{1} cuts the ground in $V^11,16$ and the **P.P.** at **16,17,18**; the mirror at **18,19** vanishing at V^1.

2_R19 is equal to **2,19**. $2_R2^1_R$ vanishes at V^1.

A B cuts the **P.P.** at **20**; the reflection of **20** is at **22** (**22,21** being equal to **20,21**).

22 V^1 contains **A B**'s reflection.

A_RD_R vanishes at V^2.

The reflection of **23** comes at **24**.

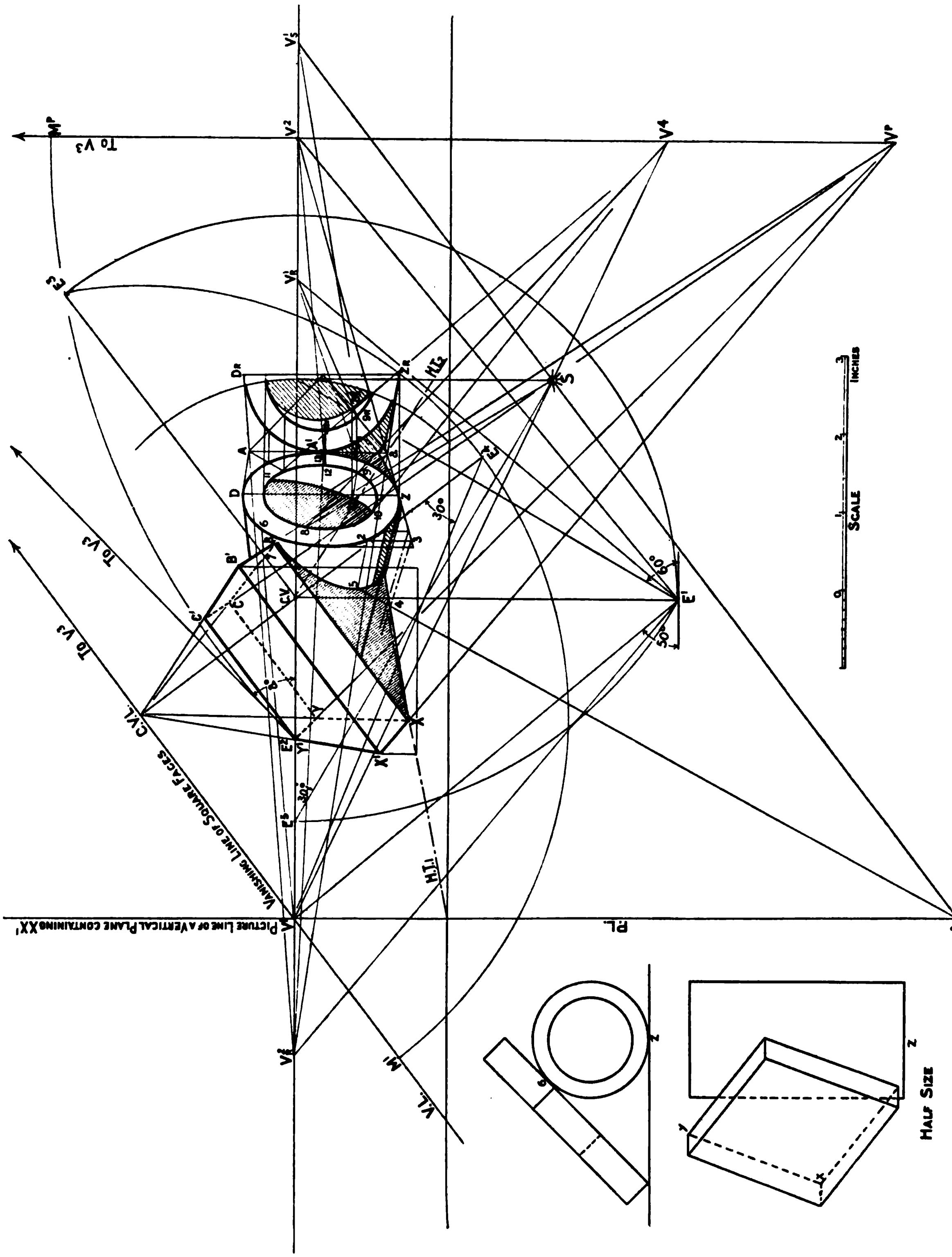
24 V^1 is the reflection of **23 X** V^1; X_R is on this line.

$V^4_RV^6$ is equal to V^4V^6.

V^5_R is as far below the horizon as V^5 is above; and V^P_R appears as high above the horizon as V^P is below.

By using these **V.P.**s the reflection of the frame may easily be obtained

PROBLEM XXXVI.



PROBLEM XXXVI.

(FROM A RECENT EXAMINATION PAPER.)

(*a*) *The diagram gives a plan and elevation of a square slab, resting against a hollow cylinder; put these objects into perspective by the use of an oblique vanishing line, in the same relative position to each other, when the nearest point* **Z** *of contact of the cylinder with the ground shall be 2 ins. below the eye, 2 ins. to the right of the centre, and $2\frac{1}{2}$ ins. within the picture, and the axis of the cylinder shall vanish horizontally towards the left at an angle of 50° to the picture plane.*

The following note was added:—*Lest any student may be weak in orthographic projection: N.B. The line* **Y X** *in the diagram is actually 4 ins. long and is inclined to the horizontal trace of the plane of the slab at an angle of 30°.*

(*b*) *Show the shadows cast by these solids when the light is coming over the left shoulder of the spectator in vertical planes at 60° to the picture plane, and vanishing downwards towards the right at 30° to the horizontal plane.*

(*c*) *Show as much of the reflection as could be seen within a visual angle of 60°, on a vertical plane perpendicular to the picture, which would touch that point in circumference of the end of the cylinder most to the right hand.*

(*a*) Place the cylinder in the given position, great care being taken in drawing the ellipses.

Find **X** on the ground in its relative position to the cylinder. The **H.T.** of the plane containing the lower face of the slab passes through **X** and vanishes at V^1, the **V.P.** of the cylinder's axis.

To obtain the **V.P.** *of inclination and the* **V.L.** *of* **X B C Y**:—E^1V^2 is the vanishing parallel of the **H.T.** of a vertical plane cutting **X B C Y** perpendicular to XV^1. V^3V^2 is the **V.L.** of this vertical plane. Rotate the eye about V^2V^3 into the **P.P.** at E^2. Measure the inclination of the slab to the ground, from the elevation; set off this inclination at E^2 (marked **a°**); this determines V^3, the required **V.P.** of inclination.

Join V^1V^3. V^1V^3 is the required **V.L.**

Rotate the eye, about V^1V^3, into the **P.P.** at E^4. Set off a line from E^4 at 30° to E^4V^1, the intersection of this line with V^1V^3 is the **V.P.** of **X Y**.

In this problem the line coincides with E^4**C.V.L.**, hence the **C.V.L.** is the **V.P.** of **X Y**, and thus **X B** will be parallel to V^1V^3.

The thickness of the slab has been measured by taking a vertical plane containing XX^1.

Complete the perspective representation as shown.

(*b*) **S** is the **V.P.** of the sun's rays (E^5 has been used in finding **S**). Determine the shadow of the end of the cylinder on the ground and the cylinder's shade line, as explained at page 67. The shadow of **X B** on the ground runs from **X** towards V^1_S, after the point **1** the shadow runs on the cylinder.

To find the shadow cast on the cylinder by a point on **X B**:—**5,2,3,**V^1 is a vertical plane cutting the cylinder parallel to its axis. **5 2** is the intersection of this plane with the cylinder; **3 4** is the same plane's **H.T.**; the shadow of **X B** would run on this plane from **4** and vanish at V^2_S (SV^2_S is parallel to V^1V^3).

5 is the intersection of V^2_S**4** with **2** V^1 and is a point on the shadow.

In a similar way find other points on the cylinder that are on the shadow of **X B** and **B C**.

N.B. **6** has been measured the same height above the ground as **6** in the elevation. **6** V^1 cuts **B C** at **7**. **7** is the point of contact of **B C** with the cylinder, and thus the shadow of **B C** on the cylinder will pass through this point.

To find the shadow cast *within* the cylinder.

From the position of the sun it will be evident that a portion of the inner rim on the left side will cast the shadow within the cylinder.

To determine the shadow cast within the cylinder by a point (as **8**):—Suppose a plane to contain the sun and a horizontal line on the cylinder passing through **8** (this line on the cylinder must vanish at V^1).

V^1S is the **V.L.** of this imaginary plane.

Determine the intersection of this plane with the vertical face of the cylinder. As V^3V^P is the **V.L.** of the vertical faces of the cylinder and V^1S is the **V.L.** of the imaginary plane containing **8**, the intersection of these **V.L.**s must be the **V.P.** of the intersection of the two planes. **8** is a point in this intersection, hence **8** V^4 is the intersection. The imaginary plane cuts the inside of the cylinder at **9** V^1. The shadow of **8** must be on **9** V^1 and on **8 S**, hence it is at 8_S.

Similarly proceed to find the shadows cast by other points.

N.B. Tangents from V^4 to circle **11,9,10** will give **10** and **11**, the limits of the shadow.

(*c*) **a** is on the ground below **A** and A^1.

C.V.a is the **H.T.** of the mirror.

As the mirror is placed vertically, and at right angles to the **P.P.**, perpendiculars to its surface will appear parallel to the **G.L.**

The reflections of AA^1 and **a** are at AA^1 and **a** respectively, V^1_R**C.V.** is equal to V^1**C.V.**, and V^2_R**C.V.** is equal to V^2**C.V.** Determine the reflections of the squares surrounding the circles, then proceed to find the reflections of the points on the curve.

The reflection of a point (**12**) is shown.

12 is a point on the inner circle. **12,13** vanishes at V^2, giving **13** on the mirror. **13,**12_R vanishes at V^2_R; **12,**12_R is horizontal and determines 12_R, the reflection of the shadow.

8_S9 vanishes at V^1; **9** is on the inner circle.

A horizontal line **9,**9_R gives 9_R on the reflection of the inner circle. The reflection of **9,**8_S vanishes at V^1_R. 8_S8_{SR} is horizontal, and thus 8_{SR} is determined (8_{SR} is the reflection of the shadow cast by **8**).

The problem is completed by determining the reflection of the shadow cast by the cylinder on the ground, but this should not require any further explanation.

EXERCISES.

(PROBLEMS TAKEN FROM A RECENT EXAMINATION PAPER.)

In working the following problems the scale to be used is ½ in. to 1 ft., and the eye is 10 ft. from the picture.

PROBLEM XXXVII.

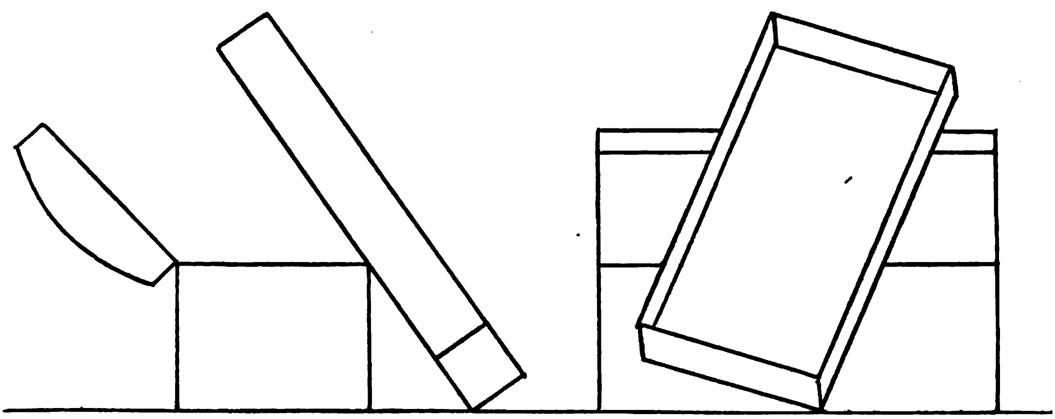
(*a*) Put into perspective an equilateral triangular prism on a horizontal plane 5 ft. below the eye, with the long edges of the prism inclined at 50° to the picture towards the right, the nearest angle being 3 ft. to the left of the centre and 4½ ft. within the picture; against the upper edge of this prism let a face of a cube be resting with one of its angles on the ground plane at a point 2½ ft. to the right of the centre and 10 ft. within the picture, and make the lowest edge of this cube to slope upwards towards the picture plane at an angle of 10° to the horizontal trace of the plane of the face of the cube which rests against the prism.

Make the faces of the prism 10 ft. long by 6 ft. wide, and the cube of 6 ft. edge.

(*b*) Cast the shadow of the cube upon the prism, and of both upon the ground plane when the light is parallel to the picture plane and inclined at 60° to the horizon downward to the left.

(*c*) Show the reflection of these solids as if they were placed upon a horizontal reflecting surface.

HALF SIZE.



PROBLEM XXXVIII.

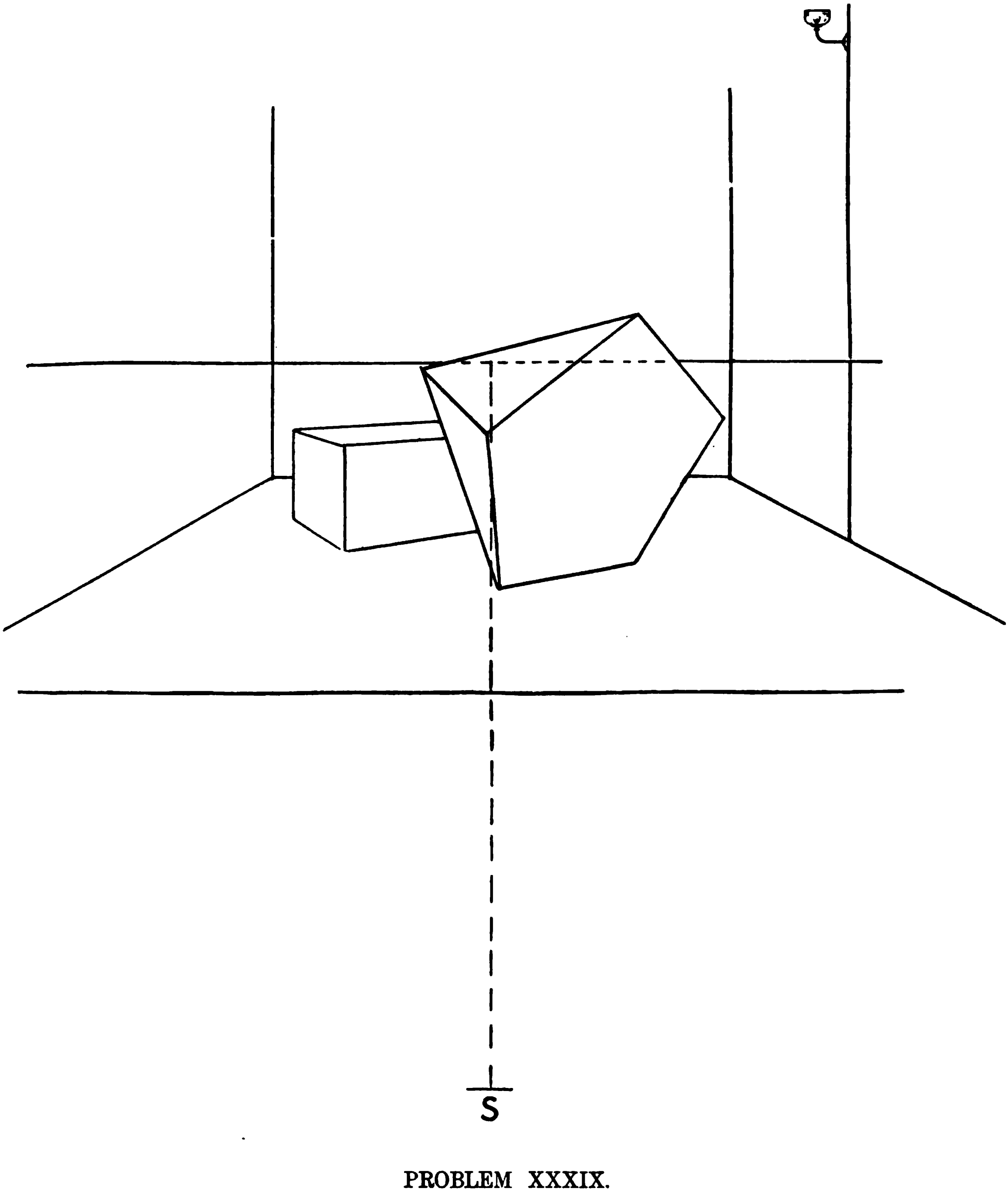
(*a*) Problem XXXVIII gives side and front elevations of a travelling trunk, with its tray taken out and standing upon the ground resting against the front edge of the trunk, thicknesses being neglected.

Put this trunk and tray in perspective, on a ground plane 5 ft. below the eye, in the same relative positions, with the nearest lower corner of the trunk opposite the spectator and 3 ft. within the picture, and its longer edges vanishing at 45° to the picture plane towards the right.

(*b*) Cast the shadows of the tray and trunk, when the light is coming over the right shoulder of the spectator, inclined at 60° to the horizontal plane, in vertical planes at 60° to the picture towards the left.

(*c*) Give the reflections, as far as they can be seen, on the supposition that the ground plane is a reflecting surface.

FULL SIZE.



PROBLEM XXXIX.

Problem XXXIX gives a perspective representation of two geometric solids grouped together; show the shadows cast by these solids—

(*a*) By sunlight when the rays are inclined at 30° to the ground plane downwards to the left in vertical planes at 45° to the picture plane.

(*b*) By an artificial light coming from the gas bracket indicated.

6.2

6.2